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Introduction to Numerical Analysis

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Preface

This book is based on a one-year introductory course on numerical analysis given by the authors at several universities in Germany and the United States. The authors concentrate on methods which can be worked out on a digital computer. For important topics, algorithmic descriptions (given more or less formally in ALGOL 60), as well as thorough but concise treatments of their theoretical foundations, are provided. Where several methods for solving a problem are presented, comparisons of their applicability and limitations are offered. Each comparison is based on operation counts, theoretical properties such as convergence rates, and, more importantly, the intrinsic numerical properties that account for the reliability or unreliability of an algorithm. Within this context, the introductory chapter on error analysis plays a special role because it precisely describes basic concepts, such as the numerical stability of algorithms, that are indispensable in the thorough treatment of numerical questions.

The remaining seven chapters are devoted to describing numerical methods in various contexts. In addition to covering standard topics, these chapters encompass some special subjects not usually found in introductions to numerical analysis. Chapter 2, which discusses interpolation, gives an account of modern fast Fourier transform methods. In Chapter 3, extrapolation techniques for speeding up the convergence of discretization methods in connection with Romberg integration are explained at length.

The following chapter on solving linear equations contains a description of a numerically stable realization of the simplex method for solving linear programming problems. Further minimization algorithms for solving unconstrained

minimization problems are treated in Chapter 5, which is devoted to solving nonlinear equations.

After a long chapter on eigenvalue problems for matrices, Chapter 7 is devoted to methods for solving ordinary differential equations. This chapter contains a broad discussion of modern multiple shooting techniques for solving two-point boundary-value problems. In contrast, methods for partial differential equations are not treated systematically. The aim is only to point out analogies to certain methods for solving ordinary differential equations, e.g., difference methods and variational techniques. The final chapter is devoted to discussing special methods for solving large sparse systems of linear equations resulting primarily from the application of difference or finite element techniques to partial differential equations. In addition to iteration methods, the conjugate gradient algorithm of Hestenes and Stiefel and the Buneman algorithm (which provides an example of a modern direct method for solving the discretized Poisson problem) are described.

Within each chapter numerous examples and exercises illustrate the numerical and theoretical properties of the various methods. Each chapter concludes with an extensive list of references.

The authors are indebted to many who have contributed to this introduction into numerical analysis. Above all, we gratefully acknowledge the deep influence of the early lectures of F. L. Bauer on our presentation. Many colleagues have helped us with their careful reading of manuscripts and many useful suggestions. Among others we would like to thank are C. Reinsch, M. B. Spijker, and, in particular, our indefatigable team of translators, R. Bartels, W. Gautschi, and C. Witzgall. Our co-workers K. Butendeich, G. Schuller, J. Zowe, and I. Brugger helped us to prepare the original German edition. Last but not least we express our sincerest thanks to Springer-Verlag for their good cooperation during the past years.

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