Daniele Funaro

Spectral Elements for Transport-Dominated Equations

With 97 Figures and 12 Tables

Springer

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