Algebraic Topology

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PREFACE TO THE SECOND SPRINGER PRINTING

IN THE MORE THAN TWENTY YEARS SINCE THE FIRST APPEARANCE OF *Algebraic Topology* the book has met with favorable response both in its use as a text and as a reference. It was the first comprehensive treatment of the fundamentals of the subject. Its continuing acceptance attests to the fact that its content and organization are still as timely as when it first appeared. Accordingly it has not been revised.

Many of the proofs and concepts first presented in the book have become standard and are routinely incorporated in newer books on the subject. Despite this, *Algebraic Topology* remains the best complete source for the material which every young algebraic topologist should know. Springer-Verlag is to be commended for its willingness to keep the book in print for future topologists.

For the current printing all of the misprints known to me have been corrected and the bibliography has been updated.

Berkeley, California
December 1989

Edwin H. Spanier

PREFACE

THIS BOOK IS AN EXPOSITION OF THE FUNDAMENTAL IDEAS OF ALGEBRAIC topology. It is intended to be used both as a text and as a reference. Particular emphasis has been placed on naturality, and the book might well have been titled *Functorial Topology.* The reader is not assumed to have prior knowledge of algebraic topology, but he is assumed to know something of general topology and algebra and to be mathematically sophisticated. Specific prerequisite material is briefly summarized in the Introduction.

Since *Algebraic Topology* is a text, the exposition in the earlier chapters is a good deal slower than in the later chapters. The reader is expected to develop facility for the subject as he progresses, and accordingly, the further he is in the book, the more he is called upon to fill in details of proofs. Because it is also intended as a reference, some attempt has been made to include basic concepts whether they are used in the book or not. As a result, there is more material than is usually given in courses on the subject.

The material is organized into three main parts, each part being made up of three chapters. Each chapter is broken into several sections which treat individual topics with some degree of thoroughness and are the basic organizational units of the text. In the first three chapters the underlying theme is the fundamental group. This is defined in Chapter One, applied in Chapter Two in the study of covering spaces, and described by means of generators and relations in Chapter Three, where polyhedra are introduced. The concept of functor and its applicability to topology are stressed here to motivate interest in the other functors of algebraic topology.

Chapters Four, Five, and Six are devoted to homology theory. Chapter Four contains the first definitions of homology, Chapter Five contains further algebraic concepts such as cohomology, cup products, and cohomology operations, and Chapter Six contains a study of topological manifolds. With each new concept introduced applications are presented to illustrate its utility.

The last three chapters study homotopy theory. Basic facts about homotopy groups are considered in Chapter Seven, applications to obstruction theory are presented in Chapter Eight, and some computations of homotopy groups of spheres are given in Chapter Nine. Main emphasis is on the application to geometry of the algebraic tools introduced earlier.

There is probably more material than can be covered in a year course. The core of a first course in algebraic topology is Chapter Four. This contains elementary facts about homology theory and some of its most important applications. A satisfactory one-semester first course for graduate students can be based on the first four chapters, either omitting or treating briefly Secs. 5 and 6 of Chapter One, Secs. 7 and 8 of Chapter Two, Sec. 8 of Chapter Three, and Sec. 8 of Chapter Four. A second one-semester course can be based on Chapters Five, Six, Seven, and Eight or on Chapters Five, Seven, Eight, and Nine. For students with knowledge of homology theory and related algebraic concepts a course in homotopy theory based on the last three chapters is quite feasible.

Each chapter is followed by a collection of exercises. These are grouped into sets, each set being devoted to a single topic or a few related topics. With few exceptions, none of the exercises is referred to in the body of the text or in the sequel. There are various types of exercises. Some are examples of the general theory developed in the preceding chapter, some treat special cases of general topics discussed later, and some are devoted to topics not discussed in the text at all. There are routine exercises as well as more difficult ones, the latter frequently with hints of how to attack them. Occasionally a topic related to material in the text is developed in a set of exercises devoted to it.

Examples in the text are usually presented with little or no indication of why they have the stated properties. This is true both of examples illustrating new concepts and of counterexamples. The verification that an example has the desired properties is left to the reader as an exercise.

The symbol ■ is used to denote the end of a proof. It is also used at the end of a statement whose proof has been given before the statement or which follows easily from previous results. Bibliographical references are by footnotes in the text. Items in each section and in each exercise set are numbered consecutively in a single list. References to items in a different section are by triples indicating, respectively, the chapter, the section or exercise set, and the number of the item in the section. Thus 3.2.2 is item 2 in Sec. 2 of Chapter Three (and 3.2 of the Introduction is item 2 in Sec. 3 of the Introduction).

The idea of writing this book originated with the existence of lecture notes based on two courses I gave at the University of Chicago in 1955. It is a pleasure to acknowledge here my indebtedness to the authors of those notes, Guido Weiss for notes of the first course, and Edward Halpern for notes of the second course. In the years since then, the subject has changed substantially and my plans for the book changed along with it, so that the present volume differs in many ways from the original notes.

The final manuscript and galley proofs were read by Per Holm. He made a number of useful suggestions which led to improvements in the text. For his comments and for his friendly encouragement at dark moments, I am sincerely grateful to him. The final manuscript was typed by Mrs. Ann Harrington and Mrs. Ollie Cullers, to both of whom I express my thanks for their patience and cooperation.

I thank the Air Force Office of Scientific Research for a grant enabling me to devote all my time during the academic year 1962–63 to work on this book. I also thank the National Science Foundation for supporting, over a period of years, my research activities some of which are discussed here.

Edwin H. Spanier

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