Undergraduate Texts in Mathematics

Editors
F. W. Gehring
P. R. Halmos

Advisory Board
C. DePrima
I. Herstein

Undergraduate Texts in Mathematics

Apostol: Introduction to Analytic Number Theory.
1976. xii, 338 pages. 24 illus.

Armstrong: Basic Topology.
1983. xii, 260 pages. 132 illus.

Bak/Newman: Complex Analysis.
1982. x, 224 pages. 69 illus.

Banchoff/Wermer: Linear Algebra Through Geometry.
1983. x, 257 pages. 81 illus.

Childs: A Concrete Introduction to Higher Algebra.
1979. xiv, 338 pages. 8 illus.

Chung: Elementary Probability Theory with Stochastic Processes.
1975. xvi, 325 pages. 36 illus.

Croom: Basic Concepts of Algebraic Topology.
1978. x, 177 pages. 46 illus.

Fischer: Intermediate Real Analysis.
1983. xiv, 770 pages. 100 illus.

Fleming: Functions of Several Variables. Second edition.
1977. xi, 411 pages. 96 illus.

Foulds: Optimization Techniques: An Introduction.
1981. xii, 502 pages. 72 illus.

Franklin: Methods of Mathematical Economics. Linear and Nonlinear Programming. Fixed-Point Theorems.
1980. x, 297 pages. 38 illus.

Halmos: Finite-Dimensional Vector Spaces. Second edition.
1974. viii, 200 pages.

Halmos: Naive Set Theory.
1974, vii, 104 pages.

Iooss/Joseph: Elementary Stability and Bifurcation Theory.
1980. xv, 286 pages. 47 illus.

Kemeny/Snell: Finite Markov Chains.
1976. ix, 224 pages. 11 illus.

Lang: Undergraduate Analysis
1983. xiii, 545 pages. 52 illus.

Lax/Burstein/Lax: Calculus with Applications and Computing, Volume 1.
1976. xi, 513 pages. 170 illus.

LeCuyer: College Mathematics with A Programming Language.
1978. xii, 420 pages. 144 illus.

Macki/Strauss: Introduction to Optimal Control Theory.
1981. xiii, 168 pages. 68 illus.

continued after Index

Kennan T. Smith

Primer of Modern Analysis

(Directions for Knowing All Dark Things,
Rhind Papyrus, 1800 B.C.)

Springer Science+Business Media, LLC

Kennan T. Smith
Mathematics Department
Oregon State University
Corvallis, Oregon 97331
U.S.A.

Editorial Board

P. R. Halmos
Indiana University
Department of Mathematics
Bloomington, Indiana 47405
U.S.A.

F. W. Gehring
University of Michigan
Department of Mathematics
Ann Arbor, Michigan 48104
U.S.A.

AMS Subject Classification: 26-01, 28-01

Library of Congress Cataloging in Publication Data
Smith, Kennan T., 1926–
Primer of modern analysis.
(Undergraduate texts in mathematics)
Includes index.
1. Mathematical analysis. I. Title. II. Series.
QA300.S77 1983 515 83-538

The original version of this book was published by Bogden & Quigley, Inc., Publishers, in 1971.

©1971 by Bogden & Quigley, Inc., Publishers.
©1983 by Springer Science+Business Media New York
Originally published by Springer-Verlag Berlin Heidelberg New York Tokyo in 1983
Softcover reprint of the hardcover 2nd edition 1983
All rights reserved. No part of this book may be translated or reproduced in any form without written permission from Springer-Verlag, 175 Fifth Avenue, New York, N.Y. 10010, U.S.A.

ISBN 978-1-4612-7021-8 ISBN 978-1-4612-1144-0 (eBook)
DOI 10.1007/978-1-4612-1144-0

To J.

Preface

This book discusses some of the first principles of modern analysis. It can be used for courses at several levels, depending upon the background and ability of the students.

It was written on the premise that today's good students have unexpected enthusiasm and nerve. When hard work is put to them, they work harder and ask for more. The honors course (at the University of Wisconsin) which inspired this book was, I think, more fun than the book itself. And better. But then there is acting in teaching, and a typewriter is a poor substitute for an audience. The spontaneous, creative disorder that characterizes an exciting course becomes silly in a book. To write, one must cut and dry. Yet, I hope enough of the spontaneity, enough of the spirit of that course, is left to enable those using the book to create exciting courses of their own.

Exercises in this book are not designed for drill. They are designed to clarify the meanings of the theorems, to force an understanding of the proofs, and to call attention to points in a proof that might otherwise be overlooked. The exercises, therefore, are a real part of the theory, not a collection of side issues, and as such nearly all of them are to be done. Some drill is, of course, necessary, particularly in the calculation of integrals.

Those using the book should not feel obliged to do every proof. It is more important for teachers to explain the theorems well and to show how they are used, and why they are interesting, than to spend all the time on proofs. This is one place where the teacher has an advantage over the author. He can choose proofs that seem to him exciting or illuminating, and skip some of the others. The author, however, must do nearly all. In this book I have omitted only the proof of Fubini's theorem—in favor of a long list of applications.

Many topics in the mathematics curriculum find their best use in the calculus of several variables: for example, much linear algebra, much topology, much measure theory, and so forth. Usually students learn them as separate topics. As a result, they understand these subjects narrowly and apply them poorly. I have therefore done quite a bit of linear algebra, topology, and measure theory—but always with the applications in mind and following close behind. The result should be that students will understand *both* sides much better.

Part I begins with a half intuitive–half rigorous discussion of applications, chosen to arouse interest and to show the need for a precise and general theory, and then develops this theory for functions of one variable. Unusual features include the solid treatment of Taylor's formula, the discussion of real analytic functions, and the Weierstrass approximation theorem.

In Part II the differential properties of functions of several variables are studied. There is some background on metric and vector spaces, but the bulk of this part deals with applications of the implicit-function theorem to the study of surfaces and manifolds, tangent and normal planes, maximum and minimum problems in several variables and on manifolds, and so forth. Various interesting sidelights, such as the derivation of Kepler's laws of planetary motion and mini–max descriptions of eigenvalues, are included.

In Part III the integration and differentiation of measures are studied. The Lebesgue theory of integration is developed in the simple, yet perfectly general, abstract setting of outer measures, and applied in many and diverse situations, such as integration in $\mathbf{R}^n$, summation of multiple power series, and Sard's theorem on regular values of differentiable functions. The Lebesgue theory of differentiation is presented for regular Borel measures on $\mathbf{R}^n$ and used, for example, in establishing the formulas for change of variable in multiple integrals. The theory of differentiation leads naturally to the study of surface area via the area measures of Hausdorff. In the final chapter I discuss the Brouwer degree of maps of spheres and its applications, developing the degree from the analytic point of view suggested by John Milnor.

Theorems, Definitions, etc., are numbered within each chapter and section. Thus, Theorem 6.3 of Chapter 8 is found in Section 6 of Chapter 8. Theorem 6.3 without any chapter reference is found in Section 6 of the chapter in which the reference is made. The chapter number and title are printed in the upper left-hand corner of each double-page spread.

The index lists most of the terms and symbols that are used and the page or pages on which they are defined. The symbols occur ahead of the terms beginning with the same letter. Thus, $|A|$ and α_m occur at the head of the a's.

I wish to thank my colleagues at Oregon State University and at the University of Oregon who read and commented upon earlier versions of the manuscript. These include Professors P. M. Anselone, D. S. Carter, R. B. Guenther, B. Petersen, and, particularly, R. M. Koch. Professor Norton Starr of Amherst College also read an earlier version of the manuscript and made suggestions. In addition, I wish to thank Professor D. C. Rung of The Pennsylvania State University for suggesting the title. Finally, I wish to praise Mr. Edward J. Quigley, who is a new publisher, but a good one.

It is fitting to end this preface with advice to the reader from the creator and patron saint of calculus. The following statement came in answer to the question of how he had made his famous discoveries:

Isaac Newton

"By always thinking about them, I keep the subject constantly before me and wait till the first dawnings open little by little into the full light."

K. T. S.

PREFACE TO THE SPRINGER EDITION

Rademacher's theorem on the differentiability of Lipschitz functions has been added. Applications of Rademacher's theorem and the Brouwer degree to changes of variable in multiple integrals have been added. The main addition, however, is a chapter on the results of Hestenes, Seeley, and Adams–Aronszajn–Smith on extension of differentiable functions of various kinds across Lipschitz graphs. A construction is given for a single extension operator which applies to functions of class C^m, functions of class C^m with bounded derivatives, functions of class C^m with Hölder continuous derivatives, and to Sobolev functions. It applies to many other function classes as well, but these are the ones discussed explicitly. The discussion of the Sobolev spaces requires a minimal knowledge of L^p spaces (mainly the Hölder and Minkowski inequalities). The theorems cover polyhedral domains, so they are of use in the numerical study of partial differential equations, as well as of theoretical interest.

K. T. S.

Contents