

Undergraduate Texts in Mathematics

Editors

F. W. Gehring

P. R. Halmos

Advisory Board

C. DePrima

I. Herstein

J. Kiefer

W. LeVeque

I. M. Singer

J. A. Thorpe

Lecture Notes on
Elementary Topology
and Geometry



Springer-Verlag

New York Heidelberg Berlin

I. M. Singer
Department of Mathematics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

J. A. Thorpe
Department of Mathematics
SUNY at Stony Brook
Stony Brook, New York 11790

Editorial Board

F. W. Gehring
Department of Mathematics
University of Michigan
Ann Arbor, Michigan 48104

P. R. Halmos
Department of Mathematics
University of California
Santa Barbara, California 93106

AMS Subject Classifications: 50-01, 53-01, 54-01

Library of Congress Cataloging in Publication Data

Singer, Isadore Manuel, 1924–
Lecture notes on elementary topology and geometry.
(Undergraduate texts in mathematics)
Reprint of the ed. published by Scott, Foresman,
Glenview, Ill.
Bibliography: p. 230
Includes index.
1. Topology. 2. Algebraic topology.
3. Geometry, Differential. I. Thorpe, John A., joint author. II. Title.
[QA611.S498 1976] 514 76–26137

All rights reserved.

No part of this book may be translated or reproduced in any form without written permission from Springer-Verlag.

© 1967 by I. M. Singer and John A. Thorpe.
Softcover reprint of the hardcover 1st edition 1967

ISBN 978-1-4615-7349-4
DOI 10.1007/978-1-4615-7347-0

ISBN 978-1-4615-7347-0 (eBook)

Preface

At the present time, the average undergraduate mathematics major finds mathematics heavily compartmentalized. After the calculus, he takes a course in analysis and a course in algebra. Depending upon his interests (or those of his department), he takes courses in special topics. If he is exposed to topology, it is usually straightforward point set topology; if he is exposed to geometry, it is usually classical differential geometry. The exciting revelations that there is some unity in mathematics, that fields overlap, that techniques of one field have applications in another, are denied the undergraduate. He must wait until he is well into graduate work to see interconnections, presumably because earlier he doesn't know enough.

These notes are an attempt to break up this compartmentalization, at least in topology-geometry. What the student has learned in algebra and advanced calculus are used to prove some fairly deep results relating geometry, topology, and group theory. (De Rham's theorem, the Gauss-Bonnet theorem for surfaces, the functorial relation of fundamental group to covering space, and surfaces of constant curvature as homogeneous spaces are the most noteworthy examples.)

In the first two chapters the bare essentials of elementary point set topology are set forth with some hint of the subject's application to functional analysis. Chapters 3 and 4 treat fundamental groups, covering spaces, and simplicial complexes. For this approach the authors are indebted to E. Spanier. After some preliminaries in Chapter 5 concerning the theory of manifolds, the De Rham theorem (Chapter 6) is proven as in H. Whitney's *Geometric Integration Theory*. In the two final chapters on Riemannian geometry, the authors follow E. Cartan and S. S. Chern. (In order to avoid Lie group theory in the last two chapters, only oriented 2-dimensional manifolds are treated.)

Preface

These notes have been used at M.I.T. for a one-year course in topology and geometry, with prerequisites of at least one semester of modern algebra and one semester of advanced calculus “done right.” The class consisted of about seventy students, mostly seniors. The ideas for such a course originated in one of the author’s tour of duty for the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America. A program along these lines, but more ambitious, can be found in the CUPM pamphlet “Pregraduate Preparation of Research Mathematicians” (1963). (See Outline III on surface theory, pp. 68–70.) The authors believe, however, that in lecturing to a large class without a textbook, the material in these notes was about as much as could be covered in a year.

Contents

Chapter 1	
Some point set topology	1
1.1 Naive set theory	1
1.2 Topological spaces	5
1.3 Connected and compact spaces	11
1.4 Continuous functions	13
1.5 Product spaces	16
1.6 The Tychonoff theorem	20
Chapter 2	
More point set topology	26
2.1 Separation axioms	26
2.2 Separation by continuous functions	31
2.3 More separability	34
2.4 Complete metric spaces	40
2.5 Applications	43
Chapter 3	
Fundamental group and covering spaces	49
3.1 Homotopy	49
3.2 Fundamental group	52
3.3 Covering spaces	62

Contents

Chapter 4

Simplicial complexes 78

- 4.1 Geometry of simplicial complexes 79
- 4.2 Barycentric subdivisions 83
- 4.3 Simplicial approximation theorem 90
- 4.4 Fundamental group of a simplicial complex 94

Chapter 5

Manifolds 109

- 5.1 Differentiable manifolds 109
- 5.2 Differential forms 118
- 5.3 Miscellaneous facts 132

Chapter 6

Homology theory and the De Rham theory 153

- 6.1 Simplicial homology 153
- 6.2 De Rham's theorem 161

Chapter 7

Intrinsic Riemannian geometry of surfaces 175

- 7.1 Parallel translation and connections 175
- 7.2 Structural equations and curvature 184
- 7.3 Interpretation of curvature 190
- 7.4 Geodesic coordinate systems 198
- 7.5 Isometries and spaces of constant curvature 207

Chapter 8

Imbedded manifolds in R^3 216

Bibliography 230

Index 231