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Lecture Notes on Elementary Topology and Geometry

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Preface

At the present time, the average undergraduate mathematics major finds mathematics heavily compartmentalized. After the calculus, he takes a course in analysis and a course in algebra. Depending upon his interests (or those of his department), he takes courses in special topics. If he is exposed to topology, it is usually straightforward point set topology; if he is exposed to geometry, it is usually classical differential geometry. The exciting revelations that there is some unity in mathematics, that fields overlap, that techniques of one field have applications in another, are denied the undergraduate. He must wait until he is well into graduate work to see interconnections, presumably because earlier he doesn't know enough.

These notes are an attempt to break up this compartmentalization, at least in topology-geometry. What the student has learned in algebra and advanced calculus are used to prove some fairly deep results relating geometry, topology, and group theory. (De Rham's theorem, the Gauss-Bonnet theorem for surfaces, the functorial relation of fundamental group to covering space, and surfaces of constant curvature as homogeneous spaces are the most noteworthy examples.)

In the first two chapters the bare essentials of elementary point set topology are set forth with some hint of the subject's application to functional analysis. Chapters 3 and 4 treat fundamental groups, covering spaces, and simplicial complexes. For this approach the authors are indebted to E. Spanier. After some preliminaries in Chapter 5 concerning the theory of manifolds, the De Rham theorem (Chapter 6) is proven as in H. Whitney's *Geometric Integration Theory*. In the two final chapters on Riemannian geometry, the authors follow E. Cartan and S. S. Chern. (In order to avoid Lie group theory in the last two chapters, only oriented 2-dimensional manifolds are treated.)

These notes have been used at M.I.T. for a one-year course in topology and geometry, with prerequisites of at least one semester of modern algebra and one semester of advanced calculus "done right." The class consisted of about seventy students, mostly seniors. The ideas for such a course originated in one of the author's tour of duty for the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America. A program along these lines, but more ambitious, can be found in the CUPM pamphlet "Pregraduate Preparation of Research Mathematicians" (1963). (See Outline III on surface theory, pp. 68–70.) The authors believe, however, that in lecturing to a large class without a textbook, the material in these notes was about as much as could be covered in a year.

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