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Best Approximation in Normed Linear Spaces by Elements of Linear Subspaces

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This monograph is a translation of the original Romanian version
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PREFACE

The existing monographs on approximation theory and on the constructive theory of functions (e.g. N. I. Ahiezer [1], J. R. Rice [192], A. F. Timan [250], I. P. Natanson [158] etc.), treat the problems of best approximation in classical style, with the methods and language of the theory of functions, the use of functional analysis being reduced to a few elementary results on best approximation in normed linear spaces and in Hilbert spaces. In contrast to these, the present monograph attempts to give a modern theory of best approximation, using in a consequent manner the methods of functional analysis.

From the vast field of best approximation, this monograph presents in more detail the results on best approximation in normed linear spaces by elements of linear subspaces, which today constitute a unified theory. The more general problems of best approximation are exposed, briefly, in Appendices I and II. A glance at the table of contents shows that together with results in general normed linear spaces there are given many applications of them in various concrete spaces.

In order to limit the size of the present monograph, we deliberately have omitted some problems related to those treated herein (e.g. applications to extremal problems of the theory of analytic functions, methods of computation of elements of best approximation, the problem of moments, connections with linear programming, etc.). Also, some results related to those presented here are mentioned without proof.

Being the first of this kind in the literature, the present monograph is based exclusively on papers published in mathematical *journals; the references to these are given in the text after each result. The bibliography given at the end does not aim at being complete, it includes only papers which are effectively quoted in the text.*

The present monograph is intended for a large circle of mathematicians. Firstly, it is addressed to specialists in approximation theory and the constructive theory of functions, offering to them the methods of functional analysis for the study of these classical domains of mathematics; the necessity and advantages of these methods are shown in the "Introduction". Secondly, it is addressed to those working in functional analysis, offering them an important field of applications. Also, taking into account that in the problems investigated there are combined the methods of functional analysis, geometry, general topology, measure theory and other mathematical disciplines, we hope that the present monograph will be useful to other categories of readers as well (e.g. to specialists in the geometry of convex bodies, etc.).

The reader is assumed to know the elements of functional analysis and the mathematical disciplines mentioned above, e.g. within the limits of university courses. However, in order to facilitate the reading and to make the book accessible to University students as well, we have indicated, in connection with the results used (of functional analysis, theory of measure, etc.), a reference to a treatise containing the proof of the respective result; when we have used results which are not to be found in monographs but only in papers published in journals, we have mentioned them in the form of lemmas, giving also their proof.

In conclusion, I wish to express my thanks to Miron Nicolescu, member of the Academy, for the invitation to write this monograph and for the constant interest shown during its elaboration. Also, I extend my thanks to my colleagues N. Dinculeanu and C. Foiaş for valuable discussions on the proofs of certain theorems and to V. Klee of the University of Washington in Seattle for some bibliographical indications.

Bucharest, July 1, 1966

THE AUTHOR

PREFACE TO THE ENGLISH EDITION

This is a translation of the original Romanian monograph, with a number of corrections of misprints and errors. I wish to express my thanks to those friends and colleagues, G. Godini (Bucharest), J. Blatter and G. Pantelidis (Bonn), A. Garkavi (Moscow), M. I. Kadec (Harkov), G. Alexits (Budapest) and others, who have called my attention to some of these corrections.

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