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Ya. G. Sinai (Ed.)

# Dynamical Systems II

Ergodic Theory  
with Applications to Dynamical Systems  
and Statistical Mechanics

With 25 Figures



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## Preface

Each author who took part in the creation of this issue intended, according to the idea of the whole edition, to present his understanding and impressions of the corresponding part of ergodic theory or its applications. Therefore the reader has an opportunity to get both concrete information concerning this quickly developing branch of mathematics and an impression about the variety of styles and tastes of workers in this field.

Ya. G. Sinai

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