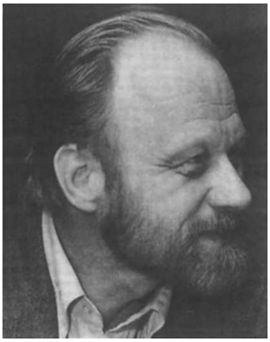
Classics in Mathematics

C. L. Siegel J. K. Moser Lectures on Celestial Mechanics

Carl Ludwig Siegel was born on December 31, 1896 in Berlin. He studied mathematics and astronomy in Berlin and Göttingen and held chairs at the Universities of Frankfurt and Göttingen before moving to the Institute for Advanced Study in Princeton in 1940. He returned to Göttingen in 1951 and died there in 1981.

Siegel was one of the leading mathematicians of the twentieth century, whose work, noted for its depth as well as breadth, ranged over many different fields such as number theory from the analytic, algebraic and geometrical points of view, automorphic functions of several complex variables, symplectic geometry, celestial mechanics.



Jürgen Moser was born on July 4, 1928 in Königsberg, then Germany. After the war he studied in Göttingen, where he received his doctoral degree in 1952 and subsequently was assistant to C. L. Siegel. In 1955 he emigrated to the USA. He held positions at M.I.T., Cambridge and primarily at the Courant Institute of Mathematical Sciences in New York; from 1967 to 1970 he was Director of this institute. In 1980 he moved to the ETH in Zürich where he now is Director of the Mathematical Research Institute.

Moser has worked in various areas of analysis. Besides celestial mechanics and KAM theory (presented in Chapter 3 of this book) he contributed to spectral theory, partial differential equations and complex analysis.

C. L. Siegel J. K. Moser

Lectures on Celestial Mechanics

Reprint of the 1971 Edition

Springer

Carl Ludwig Siegel

Jürgen K. Moser
ETH Zentrum, Mathematik
CH-8092 Zürich
Schweiz

Originally published as Vol. 187 of the
Grundlehren der mathematischen Wissenschaften

Mathematics Subject Classification (1991):
Primary 34-02, 70-02, 70F15
Secondary 34C25, 34C35, 58F05, 70F10, 70F35, 70H05, 70H15, 70H20

ISBN 978-3-540-58656-2 ISBN 978-3-642-87284-6 (ebook)
DOI 10.1007/978-3-642-87284-6

CIP data applied for

SPIN 10485228 41/3140 - 5 4 3 2 1 0 – Printed on acid-free paper

C. L. Siegel · J. K. Moser

Lectures on Celestial Mechanics

Translation by C. I. Kalme

Springer-Verlag Berlin Heidelberg New York 1971

Carl Ludwig Siegel

Mathematisches Institut der Universität Göttingen

Jürgen K. Moser

Courant Institute of Mathematical Sciences, New York

Charles I. Kalme

University of Southern California, Los Angeles

Revised and enlarged translation of „Vorlesungen über Himmelsmechanik" by C. L. Siegel, 1956 (Grundlehren der mathematischen Wissenschaften, Band 85)

AMS Subject Classifications (1970)
Primary 34-02, 70-02, 70 F 15
Secondary 34 C 25, 34 C 35, 58 F 05, 58 F 10, 70 F 35, 70 H 05, 70 H 15, 70 H 20

ISBN 3-540-05419-7 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-05419-7 Springer-Verlag New York Heidelberg Berlin

Library of Congress Catalog Card Number 71-155595. Printed in Germany. Printing and binding: Brühlsche Universitätsdruckerei, Gießen

In Memory of Franz Rellich

Preface to the First Edition (Translation)

I have lectured on the questions in celestial mechanics treated in this work at Frankfurt on Main and Baltimore as well as again at Göttingen and Princeton, most fully in a lecture series during the winter semester of 1951/52 at Göttingen. At that time Dr. J. Moser, now in New York, prepared a careful set of notes on which this publication is based.

Not being an astronomer by profession, I have made no attempt to present anew the standard methods for the determination of orbits, for which there certainly are good texts available. My aim was rather to develop some of the ideas and results that have evolved over the period of the past 70 years in the study of solutions to differential equations in the large, in which of course applications to Hamiltonian systems and in particular the equations of motion for the three-body problem occupy an important place. Even here I did not strive for completeness, but have made a selection dictated by personal interest and the hope of stimulating the listener within the framework of a lecture.

After preliminary considerations of the transformation theory for differential equations, our aim in the first chapter is to present the important results of K. F. Sundman on the three-body problem. Although Sundman's theorems are almost 50 years old, they have become known only within a small circle and have hardly effected subsequent developments. Next to Poincaré's work in the theory of differential equations, Sundman's work, despite its specialized character, belongs perhaps among the most significant developments in this area.

Even the still older "Méthodes nouvelles de la mécanique céleste" of Poincaré have not at all had the fruitful effect on the mathematics world at large one might have hoped for in view of the richness of the work. Of the next generation it was Birkhoff who penetrated these methods most deeply, and in addition to a simplified presentation and careful proofs he also contributed interesting new theorems. His book "Dynamical systems" has been a stimulus to me and is closely related to part of the problems dealt with in the remaining two chapters of our work. In the second chapter we treat the various methods for finding periodic solutions for systems of differential equations, wherein also the fixed-point method and the related work of Birkhoff is discussed in detail. In this I have for the most part assumed that we are dealing with analytic differential equations, and have derived the results by means of suitable manipulation of power series, whereby the algebraic conclusions are as far as possible kept separate from the analytic ones. A word of justification is needed as to why the investigation has been carried out only for differential equations that do not contain the independent variable t explicitly, whereas also the case of periodic dependence on t is certainly of particular interest. However, the methods in this general case do not in principle differ from those considered here, which already exhibit all the essential difficulties.

The third chapter deals with the problem of stability, and in addition to the classical result of Liapunov contains above all a discussion of the question of convergence in connection with the normal form of analytic differential equations near an equilibrium and the expansion of the general solution in trigonometric series. It was my desire also to give a complete proof at this time of the often cited theorem of Poincaré about the divergence of these series in celestial mechanics, but in this I have not succeeded. The recurrence theorem treated at the end does not quite fit into the framework of this book, but after the disappointments preceding it, it marks a conciliatory ending.

With regard to a more detailed bibliography, one should refer to Wintner's "Analytic foundations of celestial mechanics". In keeping with the character of the present work, also the list of references at the end is certainly incomplete; it serves only to name for the reader a few supplementary works to this text. The formulas here are numbered consecutively only within the individual sections. By $(a; b)$ in the text one should understand formula (b) from § a, while the symbol $[c]$ refers to the relevant place in the bibliography.

Göttingen, October 1955 Carl L. Siegel

Preface to the English Edition

The present book represents to a large extent the translation of the German „Vorlesungen über Himmelsmechanik" by C. L. Siegel. The demand for a new edition and for an English translation gave rise to the present volume which, however, goes beyond a mere translation. To take account of recent work in this field a number of sections have been added, especially in the third chapter which deals with the stability theory. Still, it has not been attempted to give a complete presentation of the subject, and the basic organization of Siegel's original book has not been altered. The emphasis lies in the development of results and analytic methods which are based on the ideas of H. Poincaré, G. D. Birkhoff, A. Liapunov and, as far as Chapter I is concerned, on the work of K. F. Sundman and C. L. Siegel. In recent years the measure-theoretical aspects of mechanics have been revitalized and have led to new results which will not be discussed here. In this connection we refer, in particular, to the interesting book by V. I. Arnold and A. Avez on "Problèmes Ergodiques de la Mécanique Classique", which stresses the interaction of ergodic theory and mechanics.

We list the points in which the present book differs from the German text. In the first chapter two sections on the triple collision in the three-body problem have been added by C. L. Siegel. Chapter II is essentially unchanged except for the inclusion of the convergence proof for the transformation into Birkhoff's normal form of an area-preserving mapping near a hyperbolic fixed-point. The main additions have been made in Chapter III. Section 26 contains a new and simpler proof for Siegel's theorem on conformal mappings near a fixed-point. Sections 32 to 36 contain a derivation of stability theorems for systems of two degrees of freedom as well as the existence theorem for quasi-periodic solutions, which are based on the work of Kolmogorov, Arnold and the undersigned. The responsibility for the accuracy of these additions rests with the undersigned.

The careful preparation of the translation is due to C. Kalme of the University of Southern California who carried out the difficult task of keeping the gist of the original book, maintaining the accuracy and producing a clear English text. In the proofreading we wish to record the

help of R. Churchill and M. Braun. H. Rüssmann suggested a simplification in the proof of Section 33.

If it was possible to preserve the spirit of the original book, it is due to Siegel's close cooperation in reading the entire English manuscript and in checking the proofs.

Princeton, April 1971 Jürgen K. Moser

Table of Contents

Chapter One. The Three-Body Problem

Chapter Two. Periodic Solutions

Chapter Three. Stability