

Joel Smoller

Shock Waves and Reaction–Diffusion Equations

Second Edition

With 165 Illustrations



Springer Science+Business Media, LLC

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Mathematics Subject Classifications (1991): 35-02, 35L67, 35Q30, 35Q35, 73D05, 76L05

Library of Congress Cataloging in Publication Data

Smoller, Joel

Shock waves and reaction–diffusion equations / Joel Smoller, —
2nd ed.

p. cm. — (Grundlehren der mathematischen Wissenschaften :
258)

Includes bibliographical references and index.

ISBN 978-1-4612-6929-8

ISBN 978-1-4612-0873-0 (eBook)

DOI 10.1007/978-1-4612-0873-0

DM168.00

1. Shock waves. 2. Reaction–diffusion equations. I. Title.

II. Series.

QA927.S57 1994

515'.353—dc20

93-50712

Printed on acid-free paper.

© 1983, 1994 Springer Science+Business Media New York

Originally published by Springer-Verlag Berlin Heidelberg New York 1994

Softcover reprint of the hardcover 2nd edition 1994

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“With a Little Help From My Friends”

(John Lennon and Paul McCartney)

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Production coordinated by Brian Howe and managed by Henry Krell; manufacturing supervised by Vincent Scelta.

Typeset by Asco Trade Typesetting Ltd., Hong Kong

9 8 7 6 5 4 3 2 1

Contents

Acknowledgment	vii
Preface to the Second Edition	ix
Preface to the First Edition	xi
List of Frequently Used Symbols	xxi
PART I	
Basic Linear Theory	1
CHAPTER 1	
Ill-Posed Problems	3
§A. Some Examples	3
§B. Lewy's Example	7
CHAPTER 2	
Characteristics and Initial-Value Problems	13
CHAPTER 3	
The One-Dimensional Wave Equation	17
CHAPTER 4	
Uniqueness and Energy Integrals	26
CHAPTER 5	
Holmgren's Uniqueness Theorem	33
CHAPTER 6	
An Initial-Value Problem for a Hyperbolic Equation	39
CHAPTER 7	
Distribution Theory	45
§A. A Cursory View	45
§B. Fundamental Solutions	52
§C. Appendix	61

CHAPTER 8	
Second-Order Linear Elliptic Equations	64
§A. The Strong Maximum Principle	65
§B. A-Priori Estimates	70
§C. Existence of Solutions	72
§D. Elliptic Regularity	76
CHAPTER 9	
Second-Order Linear Parabolic Equations	78
§A. The Heat Equation	78
§B. Strong Maximum Principles	83
PART II	
Reaction–Diffusion Equations	91
CHAPTER 10	
Comparison Theorems and Monotonicity Methods	93
§A. Comparison Theorems for Nonlinear Equations	93
§B. Upper and Lower Solutions	96
§C. Applications	99
CHAPTER 11	
Linearization	106
§A. Spectral Theory for Self-Adjoint Operators	106
§B. Linearized Stability	114
§C. Appendix: The Krein–Rutman Theorem	122
CHAPTER 12	
Topological Methods	126
§A. Degree Theory in \mathbf{R}^n	126
§B. The Leray–Schauder Degree	139
§C. An Introduction to Morse Theory	146
§D. A Rapid Course in Topology	156
CHAPTER 13	
Bifurcation Theory	167
§A. The Implicit Function Theorem	168
§B. Stability of Bifurcating Solutions	176
§C. Some General Bifurcation Theorems	181
§D. Spontaneous Bifurcation; An Example	185
CHAPTER 14	
Systems of Reaction–Diffusion Equations	192
§A. Local Existence of Solutions	193
§B. Invariant Regions	198
§C. A Comparison Theorem	212
§D. Decay to Spatially Homogeneous Solutions	222
§E. A Lyapunov Function for Contracting Rectangles	227
§F. Applications to the Equations of Mathematical Ecology	230

PART III	
The Theory of Shock Waves	237
 CHAPTER 15	
Discontinuous Solutions of Conservation Laws	239
§A. Discontinuous Solutions	241
§B. Weak Solutions of Conservation Laws	246
§C. Evolutionary Systems	254
§D. The Shock Inequalities	259
§E. Irreversibility	261
 CHAPTER 16	
The Single Conservation Law	265
§A. Existence of an Entropy Solution	266
§B. Uniqueness of the Entropy Solution	281
§C. Asymptotic Behavior of the Entropy Solution	291
§D. The Riemann Problem for a Scalar Conservation Law	301
 CHAPTER 17	
The Riemann Problem for Systems of Conservation Laws	306
§A. The p -System	306
§B. Shocks and Simple Waves	320
§C. Solution of the General Riemann Problem	334
 CHAPTER 18	
Applications to Gas Dynamics	337
§A. The Shock Inequalities	338
§B. The Riemann Problem in Gas Dynamics	346
§C. Interaction of Shock Waves	358
 CHAPTER 19	
The Glimm Difference Scheme	368
§A. The Interaction Estimate	369
§B. The Difference Approximation	376
§C. Convergence	385
 CHAPTER 20	
Riemann Invariants, Entropy, and Uniqueness	391
§A. Riemann Invariants	392
§B. A Concept of Entropy	397
§C. Solutions with “Big” Data	403
§D. Instability of Rarefaction Shocks	414
§E. Oleinik’s Uniqueness Theorem	416
 CHAPTER 21	
Quasi-Linear Parabolic Systems	426
§A. Gradient Systems	426
§B. Artificial Viscosity	431
§C. Isentropic Gas Dynamics	435

PART IV	
The Conley Index	445
CHAPTER 22	
The Conley Index	447
§A. An Impressionistic Overview	448
§B. Isolated Invariant Sets and Isolating Blocks	458
§C. The Homotopy Index	472
CHAPTER 23	
Index Pairs and the Continuation Theorem	478
§A. Morse Decompositions and Index Pairs	479
§B. The Conley Index of an Isolated Invariant Set	486
§C. Continuation	494
§D. Some Further Remarks	502
CHAPTER 24	
Travelling Waves	507
§A. The Structure of Weak Shock Waves	508
§B. The Structure of Magnetohydrodynamic Shock Waves	514
§C. Periodic Travelling Waves	521
§D. Stability of Steady-State Solutions	530
§E. Instability of Equilibrium Solutions of the Neumann Problem	542
§F. Appendix: A Criterion for Nondegeneracy	549
CHAPTER 25	
Recent Results	553
Section I. Reaction–Diffusion Equations	553
§A. Stability of the Fitz-Hugh–Nagumo Travelling Pulse	553
§B. Symmetry-Breaking	555
§C. A Bifurcation Theorem	559
§D. Equivariant Conley Index	563
§E. Application to Semilinear Elliptic Equations	566
Concluding Remarks	568
Section II. Theory of Shock Waves	569
§A. Compensated Compactness	569
§B. Stability of Shock Waves	576
§C. Miscellaneous Results	581
Section III. Conley Index Theory	583
§A. The Connection Index	583
§B. Conley's Connection Matrix	586
§C. Miscellaneous Results	590
Section IV. Stability of Travelling Waves—A Topological Approach	590
§A. Introduction	590
§B. The Search for $\sigma_p(L)$	592
§C. Applications to Fast–Slow Systems	598
References	603
Bibliography	607
Author Index	621
Subject Index	625