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# Basic Set Theory

A. Shen

N. K. Vereshchagin



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ABSTRACT. The book is based on lectures given by the authors to undergraduate students at Moscow State University. It explains basic notions of “naive” set theory (cardinalities, ordered sets, transfinite induction, ordinals). The book can be read by undergraduate and graduate students and all those interested in basic notions of set theory. The book contains more than 100 problems of various degrees of difficulty.

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# Contents

Preface	vii
Chapter 1. Sets and Their Cardinalities	1
§1. Sets	1
§2. Cardinality	4
§3. Equal cardinalities	7
§4. Countable sets	9
§5. Cantor–Bernstein Theorem	16
§6. Cantor’s Theorem	24
§7. Functions	30
§8. Operations on cardinals	35
Chapter 2. Ordered Sets	41
§1. Equivalence relations and orderings	41
§2. Isomorphisms	47
§3. Well-founded orderings	52
§4. Well-ordered sets	56

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§5. Transfinite induction	59
§6. Zermelo's Theorem	66
§7. Transfinite induction and Hamel basis	69
§8. Zorn's Lemma and its application	74
§9. Operations on cardinals revisited	78
§10. Ordinals	83
§11. Ordinal arithmetic	87
§12. Recursive definitions and exponentiation	91
§13. Application of ordinals	99
Bibliography	109
Glossary	111
Index	113