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3

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To my wife, Karin

Preface

This edition of Books IV to VII of Diophantus' *Arithmetica*, which are extant only in a recently discovered Arabic translation, is the outgrowth of a doctoral dissertation submitted to the Brown University Department of the History of Mathematics in May 1975. Early in 1973, my thesis adviser, Gerald Toomer, learned of the existence of this manuscript in A. Gulchīn-i Ma'ānī's just-published catalogue of the mathematical manuscripts in the Mashhad Shrine Library, and secured a photographic copy of it. In September 1973, he proposed that the study of it be the subject of my dissertation. Since limitations of time compelled us to decide on priorities, the first objective was to establish a critical text and to translate it. For this reason, the Arabic text and the English translation appear here virtually as they did in my thesis. Major changes, however, are found in the mathematical commentary and, even more so, in the Arabic index. The discussion of Greek and Arabic interpolations is entirely new, as is the reconstruction of the history of the *Arithmetica* from Diophantine to Arabic times.

It is with the deepest gratitude that I acknowledge my great debt to Gerald Toomer for his constant encouragement and invaluable assistance. It was under his guidance that I learned how to read mediaeval mathematical manuscripts and how to establish a critical text. He spared neither his time nor his energy, abandoning his own scholarly pursuits in order to facilitate my study of the Diophantus manuscript. This generous help also continued after the completion of the thesis; virtually all new ideas or interpretations have undergone his scrutiny.

I should also like to thank my former professor at the Swiss Federal Institute of Technology in Zurich (ETHZ), Dr. Peter Huber, who first encouraged me to study the History of Mathematics and who later helped procure a grant for me from ETHZ. This, together with Brown University's waiving of tuition fees, enabled me to pursue my studies at Brown University for three years.

During my stay in Providence and since then, I have enjoyed the continuing help and encouragement of Professor emer. O. Neugebauer, Professor and Mrs. A. Sachs, and Professor D. Pingree. All contributed to my formation and offered many valuable suggestions for my work.

Special thanks are due to the curator of the Shrine Library in Mashhad, Aḥmad Gulchīn-i Maʿānī, for kindly having made available the necessary photographic reproductions to Gerald Toomer.

While preparing my thesis and this edition, I has occasion to consult a considerable amount of literature, and I should like to express my sincere thanks to the personnel of the Brown University Library and, more particularly, to the personnel of the Bibliothèque Publique et Universitaire of Geneva for their generous assistance.

Finally, I must express my gratitude to my wife, Karin, on whom devolved the delicate task of reading, polishing, and often reformulating the English text and many arguments in it. Despite family obligations, she found time to read the entire text several times and to rewrite the unsatisfactory parts of it until a coherent whole emerged. It is to her that I dedicate this, my first book.

Geneva,
Switzerland
September, 1982.

JACQUES SESIANO

Table of Contents

Part One: Introduction	1
Chapter I. The Four Arabic Books and the <i>Arithmetica</i>	3
§1. Authenticity of the Arabic Books	3
1. That the Arabic Books Belong to the <i>Arithmetica</i>	4
2. Concerning Their Place Within the <i>Arithmetica</i>	4
a. Placement of the Arabic Book IV	4
b. Basic Methods Used in the Arabic Books	5
c. Placement of the Four Arabic Books	7
§2. Diophantus in Islamic, and Byzantine, Times	8
1. Qusṭā ibn Lūqā and the <i>Arithmetica</i>	8
2. Islamic Mathematicians and the <i>Arithmetica</i>	9
a. Abū Kāmil	9
b. Al-Ḥazīn	10
c. Abū'l-Wafā'	10
d. Al-Karājī	10
e. Ibn al-Haiṭam	11
f. Samaw'al ibn Yaḥyā	11
Appendix. Designation of the <i>Arithmetica</i> in Arabic	13
3. Mathematicians and the <i>Arithmetica</i> in Byzantium	14
a. The Time of Leon the Mathematician	14
α. The Seventh and Eighth Centuries	14
β. The Century of Leon	14
γ. From Leon to Planudes	16
b. The Time of Maximus Planudes	16
c. Oldest Greek Manuscripts Still Extant	18
α. Non-Planudean Class	18
β. Planudean Class	20

Chapter II. The Extant Arabic Text	21
§3. Description of the Manuscript	21
§4. Orthographical Remarks	23
1. Writing of the <i>hamza</i> ^h	23
2. Particular Endings	28
3. Numerals	28
4. Repeated, Erroneous Spellings	29
§5. Additions by Earlier Readers (or Copyists)	29
§6. On the Progenitor of Our Manuscript	36
§7. Grammatical and Lexicological Remarks	37
1. Numbers and Powers	37
A. Integers	37
a. Grammatical Peculiarities	37
b. Determination	38
B. Fractions	39
a. General Fractions	39
α . Expression	39
β . Determination	40
b. Aliquot Fractions and Related Cases	40
α . Expression	40
β . Decomposition of Some Fractions	41
γ . Grammatical Peculiarities Connected with Aliquot Fractions	41
C. Grammatical Number of a Mathematical Expression	42
a. Units	42
b. Multiple of a Power	42
c. Algebraic Polynomial Expression	43
D. Powers	43
a. The Greek Power-system	43
b. The Arabic Power-system	44
c. The Power-system in Our Text	44
α . x^5	45
β . x^8	45
d. Grammatical Determination of the Powers	46
α . Two Elements	46
β . Three Elements	46
2. Some Grammatical Remarks on Verbs	46
a. Verbal Persons Used	46
b. Jussives of Weak Verbs	46
c. The Verb <i>ʿadala</i>	47
α . Agreement of <i>ʿadala</i>	47
β . Agreement of the Auxiliary of <i>ʿadala</i>	47

Chapter III. Tentative Reconstruction of the History of the <i>Arithmetica</i>	48
§8. Formal Subdivisions of a Problem	48
1. Analysis and Synthesis	48
2. Subdivisions of a Problem	49
§9. Major, Unsystematic Supplements in the <i>Arithmetica</i>	50
1. Interpolated Problems in the <i>Arithmetica</i>	51
2. Alternative Resolutions (ἄλλως)	54
a. In the Greek Books	54
b. In the Arabic Books	54
3. Other Supplements	55
a. Corollaries	55
b. Remarks	56
c. Additional Computations	56
Appendix. A Comparison Between al-Karajī's Version and the Extant <i>Arithmetica</i>	57
§10. Errors in the Problems of the Arabic Books	60
§11. Quality of the Translation	65
1. Imperfections in the Translation	65
2. General Character of the Translation	67
§12. Genealogy of the Mashhad Manuscript	68
1. Earliest Additions	68
2. The Major Commentary	68
a. Additions Originating with the Major Commentary	68
α. Additions in the Analysis	69
β. Additions in the Synthesis	69
b. Value of This Commentary	70
c. Possible Authorship of the Major Commentary	71
3. The Addition of the Final Statements	72
4. The Arabic Diophantus	73
5. Genealogical Tree of the Mashhad Manuscript	74
§13. On the Missing Part of the <i>Arithmetica</i>	76
1. New Aspects of the Problem	76
2. The Announcement in the Greek Introduction	77
3. Diophantus and the Equation $Ax^2 + Bx + C = \square$	78
4. On Some Problems of a Diophantine Nature Found in Islamic Mathematics but Not in the Extant <i>Arithmetica</i>	81
a. Problems of Abū Kāmil	81
b. Problems of al-Karajī	82
5. Conclusion	83

Part Two: Translation	85
Book IV	86
Book V	126
Book VI	139
Book VII	156
Part Three: Mathematical Commentary	173
Book IV	175
Book V	223
Book VI	244
Book VII	261
Part Four: Text	281
Book IV	283
Book V	352
Book VI	374
Book VII	403
Part Five: Arabic Index	431
Appendix: Conspectus of the Problems of the <i>Arithmetica</i>	461
Bibliography	485
General Index	493