

Graduate Texts in Mathematics **67**

*Editorial Board*

S. Axler F.W. Gehring K.A. Ribet

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## Graduate Texts in Mathematics

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*(continued after index)*

Jean-Pierre Serre

# Local Fields

Translated from the French by  
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