Graduate Texts in Mathematics

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Linear Representations of Finite Groups

Translated from the French by
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Springer-Verlag
New York Heidelberg Berlin

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AMS Subject Classification: Primary 20Cxx
Secondary 20Dxx, 12B15, 16A50, 16A54

Library of Congress Cataloging in Publication Data

Serre, Jean-Pierre.
Linear representations of finite groups.
(Graduate texts in mathematics ; 42)
Translation of Représentations linéaires des groupes finis, 2. ed.
Includes bibliographies and indexes.
1. Representations of groups. 2. Finite groups. I. Title. II. Series.
QA171.S5313 512'.2 76-12585

Translation of the French edition
Représentations linéaires des groupes finis, Paris: Hermann 1971

Softcover reprint of the hardcover 1st edition 1977

9 8 7 6 5 4 3 2 1

ISBN 978-1-4684-9460-0 ISBN 978-1-4684-9458-7 (eBook)
DOI 10.1007/978-1-4684-9458-7

Preface

This book consists of three parts, rather different in level and purpose:

The first part was originally written for quantum chemists. It describes the correspondence, due to Frobenius, between linear representations and characters. This is a fundamental result, of constant use in mathematics as well as in quantum chemistry or physics. I have tried to give proofs as elementary as possible, using only the definition of a group and the rudiments of linear algebra. The examples (Chapter 5) have been chosen from those useful to chemists.

The second part is a course given in 1966 to second-year students of l'École Normale. It completes the first on the following points:
(a) degrees of representations and integrality properties of characters (Chapter 6);
(b) induced representations, theorems of Artin and Brauer, and applications (Chapters 7–11);
(c) rationality questions (Chapters 12 and 13).
The methods used are those of linear algebra (in a wider sense than in the first part): group algebras, modules, noncommutative tensor products, semisimple algebras.

The third part is an introduction to Brauer theory: passage from characteristic 0 to characteristic p (and conversely). I have freely used the language of abelian categories (projective modules, Grothendieck groups), which is well suited to this sort of question. The principal results are:
(a) The fact that the decomposition homomorphism is surjective: all irreducible representations in characteristic p can be lifted "virtually" (i.e., in a suitable Grothendieck group) to characteristic 0.
(b) The Fong–Swan theorem, which allows suppression of the word "virtually" in the preceding statement, provided that the group under consideration is p-solvable.

I have also given several applications to the Artin representations.

I take pleasure in thanking:

Gaston Berthier and Josiane Serre, who have authorized me to reproduce Part I, written as an Appendix to their book, *Quantum Chemistry*;
Yves Balasko, who drafted a first version of Part II from some lecture notes;
Alexandre Grothendieck, who has authorized me to reproduce Part III, which first appeared in his Séminaire de Géométrie Algébrique, I.H.E.S., 1965/66.

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