

Series in Banach Spaces

Conditional and Unconditional Convergence

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Translated from the Russian by Andrei Iacob

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