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George R. Sell   Yuncheng You

Dynamics of Evolutionary Equations

With 19 Illustrations

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George R. Sell
School of Mathematics
University of Minnesota
Minneapolis, MN 55455, USA
sell@math.umn.edu

Yuncheng You
Department of Mathematics
University of South Florida
Tampa, FL 33620-5700, USA
you@math.usf.edu

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With great fondness and gratitude, we dedicate this volume to our parents.

George P. Sell and Alice O. (Roecker) Sell
and
You Qiwen and Wei Yinmei

PREFACE

The theory and applications of infinite dimensional dynamical systems have attracted the attention of scientists for quite some time. Dynamical issues arise in equations that attempt to model phenomena that change with time. The infinite dimensional aspects occur when forces that describe the motion depend on spatial variables, or on the history of the motion. In the case of spatially dependent problems, the model equations are generally partial differential equations, and problems that depend on the past give rise to differential-delay equations. Because the nonlinearities occurring in thse equations need not be small, one needs good dynamical theories to understand the longtime behavior of solutions.

Our basic objective in writing this book is to prepare an entreé for scholars who are beginning their journey into the world of dynamical systems, especially in infinite dimensional spaces. In order to accomplish this, we start with the key concepts of a *semiflow* and a *flow*. As is well known, the basic elements of dynamical systems, such as the theory of attractors and other invariant sets, have their origins here.

In the applications to partial differential equations, for example, the properties of a semiflow serve as a precise statement of the notion of a well-posed problem, which is a central feature in the study of reaction diffusion equations, nonlinear wave equations, and the Navier-Stokes equations.This concept serves as a road map for finding proper solutions in order to drive into the inner city of dynamics of partial differential equations.

Since a time-varying solution of a partial differential equation can be viewed as a trajectory, or curve, in some Banach space W, this suggests that one should rewrite the equation of motion of this solution as an equation in W. The resulting equation is called an evolutionary equation, be it linear or nonlinear. The main approach in this volume is built around the theory of evolutionary equations. (See Chapters 3 and 4.)

Chapter 4 is an especially important feature of this work. Many aspects of evolutionary equations are collected here, perhaps for the first time, in book form. One should read this chapter on two levels: as a basic introduction and as a reference source. A good approach is to read it more than once, where one goes deeper as the need arises.

The basic applications to the semiflow theory of the Navier-Stokes equations and other partial differential equations are in Chapters 5 and 6. Several aspects of the modern theories of dynamical systems to linear and nonlinear evolutionary equations, such as the perturbation theory of a saddle point, the reduction principle and center manifold, periodic orbits and invariant manifolds, and inertial manifolds appear in Chapters 7 and 8.

Chapter 1 is a brief essay on the Evolution of Evolutionary Equations, with emphasis on the theory of the longtime dynamics of the solutions of these equations. We have chosen to use some poetic license to keep this chapter short. As a result, we do not include an exhaustive list of references to the literature in this chapter. Additional references do appear in the Commentary sections elsewhere in this volume.

There are a number of general readings that are relevant. First there is the pioneering set of lecture notes by Henry (1981) on dynamical issues of nonlinear partial differential equations. Next there is the book by Temam (1988), which contains an encyclopedic treatment of many applications arising in mechanics and physics.The monograph by Hale (1988) contains valuable information on nonlinear dynamics in infinite dimensions, with applications to partial differential equations and differential equations with time delays. For a very good treatment of the dynamics of functional differential equations, see Hale and Verduyn Lunel (1993). The book by Pazy (1983) contains an excellent treatment of the linear theory of semigroups on Banach spaces. Background information on metric space theory, the geometry of Hilbert spaces, and the theory of linear operators can be found in Naylor and Sell (1982). An extensive bibliography on dynamical systems is available on the World Wide Web; see Sell (2000). The references in this bibliography are updated from time to time.

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