

Franz Schwabl

Statistical Mechanics

Translated by William Brewer

With 202 Figures, 26 Tables,
and 189 Problems



Springer

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