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Stability and Transition in Shear Flows

With 222 figures

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II Stability of Complex Flows and Transition

Preface

The field of hydrodynamic stability has a long history, going back to Reynolds and Lord Rayleigh in the late 19th century. Because of its central role in many research efforts involving fluid flow, stability theory has grown into a mature discipline, firmly based on a large body of knowledge and a vast body of literature. The sheer size of this field has made it difficult for young researchers to access this exciting area of fluid dynamics.

For this reason, writing a book on the subject of hydrodynamic stability theory and transition is a daunting endeavor, especially as *any* book on stability theory will have to follow into the footsteps of the classical treatises by Lin (1955), Betchov & Criminale (1967), Joseph (1971), and Drazin & Reid (1981). Each of these books has marked an important development in stability theory and has laid the foundation for many researchers to advance our understanding of stability and transition in shear flows.

A task every author has to face is the choice of material to include in a book, while being fully aware of the fact that full justice cannot be done to all areas. The past two decades have seen a great deal of development in hydrodynamic stability theory. For this reason we chose to devote a substantial fraction of this book to recent developments in stability theory which, among others, include nonmodal analysis, spatial growth, adjoint techniques, parabolized stability equations, secondary instability theory and direct numerical simulations. Some more classical theories are included for completeness, but are treated in less detail, especially if they are covered elsewhere in the literature. Other topics such as, critical layer theory, advanced asymptotic methods, bifurcation and chaos theory, have been omitted altogether due to space constraints. We sincerely hope that the reader will find our choice of material interesting and stimulating. Throughout the text references are provided that will guide the interested reader to more details, different applications and various extensions, but no attempt has been made to compile an exhaustive bibliography.

The book is foremost intended for researchers and graduate students with a basic knowledge of fundamental fluid dynamics. We particularly hope it will help young researchers at the beginning of their scientific careers to quickly gain an overview as well as detailed knowledge of the recent developments in the field. Various sections of the text have been used in graduate courses on hydrodynamic stability at the University of Washington, Seattle and the Royal Institute of Technology, Stockholm, Sweden.

The book consists of an introduction and two parts. The first part (Chapters 2-5) develops the fundamental concepts underlying stability theory. Chapter 2 deals with the temporal evolution of disturbances in an inviscid fluid. The linear theory for viscous fluids is developed in Chapters 3 and 4 with Chapter 3 concentrating on a modal description and Chapter 4 introducing the nonmodal framework. In Chapter 5 we discuss finite-amplitude effects and study various nonlinear stability theories.

The second part of the book (Chapters 6-9) covers more advanced topics. In Chapter 6 we will study the influence of various physical effects (such as rotation, curvature, compressibility, etc.) on the stability behavior of parallel shear flows. Chapter 7 is devoted to spatial stability theory covering such topics as absolute stability theory, weakly nonparallel effects, parabolized stability equations, and receptivity. Secondary instability theory is treated in Chapter 8 with applications to Tollmien-Schlichting waves, streaks, and vortical flows. In Chapter 9 many of the concepts introduced in the previous chapters are used to explain the transition process from laminar to turbulent fluid motion in a variety of shear flows. This chapter introduces and analyzes different transition scenarios observed in experiments and direct numerical simulations.

The appendices provide helpful hints on numerical methods, present more detailed derivations, and suggest some practice problems.

Over the course of the past years many colleagues and friends have contributed to this book through preprints of latest results, reprints of past results, insightful comments on the manuscript and moral support.

We wish to thank Larry Sirovich and Kenny Breuer who got us started on this demanding, but rewarding, project. Their comments and encouragement are appreciated greatly. We also thank Håkan Gustavsson for his contributions to an earlier review article that served as a starting point for this book.

We are deeply indebted to Alex Bottaro for his insightful comments on all parts of the book and to Philip Drazin for his careful proofreading of the manuscript. We thank Nick Trefethen for his detailed comments and his interest, encouragement and enthusiasm.

Many colleagues have generously provided comments and material from their past and current research. We especially wish to thank Henrik Alfredsson, David Ashpis, Martin Berggren, Alex Bottaro, Carlo Cossu, Bill Criminale, Jeffrey Crouch, Ardeshir Hanifi, J. Healey, Werner Koch, Rebecca Lingwood, Satish Reddy, Ulrich Rist, Michael Rogers, Jerry Swearingen, Nick Trefethen, Anatoli Tumin, and Akiva Yaglom.

A large part of the writing was done during the first author's many visits to the Department of Mechanics at the Royal Institute of Technology (KTH) in Stockholm, during the second author's visit to the Department of Applied Mathematics at the University of Washington, and during a visit to the Center for Turbulence Research at Stanford University. We would like to thank the chairs of these institutions, Arne Johansson (KTH), K.K. Tung (UW), and Parviz Moin (CTR), for their warm hospitality and support. In particular, the first author is greatly indebted to the faculty and students at KTH and will always treasure the interesting discussions, memorable activities, and Swedish hospitality. The second author gratefully acknowledges the support of the Aeronautical Research Institute of Sweden.

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Last, but certainly not least, we wish to thank our families for supporting this effort and for enduring the many sacrifices it took.

Seattle and Stockholm, July 2000

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