

Contents

<i>Prelude</i>	<i>page</i> viii
<i>Dependence chart</i>	xi
1 Prologue	1
Problems	4
2 The pleasures of counting	5
Problems	13
3 σ-algebras	15
Problems	20
4 Measures	22
Problems	28
5 Uniqueness of measures	31
Problems	35
6 Existence of measures	37
Problems	46
7 Measurable mappings	49
Problems	54
8 Measurable functions	57
Problems	65
9 Integration of positive functions	67
Problems	73
10 Integrals of measurable functions and null sets	76
Null sets and the ‘a.e.’	80
Problems	84

11 Convergence theorems and their applications	88
Parameter-dependent integrals	91
Riemann vs. Lebesgue integration	92
Examples	98
Problems	100
12 The function spaces \mathcal{L}^p, $1 \leq p \leq \infty$	105
Problems	116
13 Product measures and Fubini's theorem	120
More on measurable functions	127
Distribution functions	128
Minkowski's inequality for integrals	130
Problems	130
14 Integrals with respect to image measures	134
Convolutions	137
Problems	140
15 Integrals of images and Jacobi's transformation rule	142
Jacobi's transformation formula	147
Spherical coordinates and the volume of the unit ball	152
Continuous functions are dense in $\mathcal{L}^p(\lambda^n)$	156
Regular measures	158
Problems	159
16 Uniform integrability and Vitali's convergence theorem	163
Different forms of uniform integrability	168
Problems	173
17 Martingales	176
Problems	188
18 Martingale convergence theorems	190
Problems	200
19 The Radon–Nikodým theorem and other applications of martingales	202
The Radon–Nikodým theorem	202
Martingale inequalities	211
The Hardy–Littlewood maximal theorem	213
Lebesgue's differentiation theorem	218
The Calderón–Zygmund lemma	221
Problems	222

	<i>Contents</i>	vii
20 Inner product spaces	226	
Problems	232	
21 Hilbert space \mathcal{H}	234	
Problems	246	
22 Conditional expectations in L^2	248	
On the structure of subspaces of L^2	253	
Problems	257	
23 Conditional expectations in L^p	258	
Classical conditional expectations	263	
Separability criteria for the spaces $L^p(X, \mathcal{A}, \mu)$	269	
Problems	274	
24 Orthonormal systems and their convergence behaviour	276	
Orthogonal polynomials	276	
The trigonometric system and Fourier series	283	
The Haar system	289	
The Haar wavelet	295	
The Rademacher functions	299	
Well-behaved orthonormal systems	302	
Problems	312	
Appendix A: \liminf and \limsup	313	
Appendix B: Some facts from point-set topology	318	
Topological spaces	319	
Metric spaces	322	
Normed spaces	325	
Appendix C: The volume of a parallelepiped	328	
Appendix D: Non-measurable sets	330	
Appendix E: A summary of the Riemann integral	337	
The (proper) Riemann integral	337	
The fundamental theorem of integral calculus	346	
Integrals and limits	351	
Improper Riemann integrals	353	
<i>Further reading</i>	360	
<i>References</i>	364	
<i>Notation index</i>	367	
<i>Name and subject index</i>	371	