Mechanics

Springer-Verlag Berlin Heidelberg GmbH

Florian Scheck

# Mechanics

## From Newton's Laws to Deterministic Chaos

3rd Edition
With 145 Figures, 2 Tables and 117 Problems and Solutions

Springer

Professor
Dr. Florian A. Scheck
Institut für Physik, Theoretische Elementarteilchenphysik
Johannes Gutenberg-Universität
Staudingerweg 7
D-55099 Mainz, Germany

Title of the original German edition: *Mechanik*, 5. Auflage (Springer-Lehrbuch)
ISBN 978-3-540-65558-9


ISBN 978-3-540-65558-9

Library of Congress Cataloging-in-Publication Data applied for
Scheck, Florian: Mechanics: from Newton's laws to deterministic chaos/Florian Scheck. – 3rd ed.
Dt. Ausg. u.d.T.: Scheck, Florian: Mechanik
ISBN 978-3-540-65558-9 ISBN 978-3-662-03748-5 (eBook)
DOI 10.1007/978-3-662-03748-5


Originally published by Springer-Verlag Berlin Heidelberg New York in 1999



Typesetting: Data conversion by Frank Herweg, Hirschberg-Leutershausen
Cover picture shows Figure 1.20 on page 49 and Figure 6.3 on page 346
Cover design: *design & production* GmbH, Heidelberg

SPIN: 10712219 56/3142 - 5 4 3 2 1 0 – Printed on acid-free paper

# Preface

**Purpose and Emphasis.** Mechanics not only is the oldest branch of physics but was and still is the basis for all of theoretical physics. Quantum mechanics can hardly be understood, perhaps cannot even be formulated, without a good knowledge of general mechanics. Field theories such as electrodynamics borrow their formal framework and many of their building principles from mechanics. In short, throughout the many modern developments of physics where one frequently turns back to the principles of classical mechanics its model character is felt. For this reason it is not surprising that the *presentation* of mechanics reflects to some extent the development of modern physics and that today this classical branch of theoretical physics is taught rather differently than at the time of Arnold Sommerfeld, in the 1920s, or even in the 1950s, when more emphasis was put on the theory and the applications of partial-differential equations. Today, *symmetries* and *invariance principles*, the *structure of the space–time continuum*, and the *geometrical structure* of mechanics play an important role. The beginner should realize that mechanics is not primarily the art of describing block-and-tackles, collisions of billiard balls, constrained motions of the cylinder in a washing machine, or bicycle riding. However fascinating such systems may be, mechanics is primarily the field where one learns to develop general principles from which equations of motion may be derived, to understand the importance of symmetries for the dynamics, and, last but not least, to get some practice in using theoretical tools and concepts that are essential for all branches of physics.

Besides its role as a basis for much of theoretical physics and as a training ground for physical concepts, mechanics is a fascinating field in itself. It is not easy to master, for the beginner, because it has many different facets and its structure is less homogeneous than, say, that of electrodynamics. On a first assault one usually does not fully realize both its charm and its difficulty. Indeed, on returning to various aspects of mechanics, in the course of one's studies, one will be surprised to discover again and again that it has new facets and new secrets. And finally, one should be aware of the fact that mechanics is not a closed subject, lost forever in the archives of the nineteenth century. As the reader will realize in Chap. 6, if he or she has not realized it already, mechanics is an exciting field of research with many important questions of qualitative dynamics remaining unanswered.

**Structure of the Book and a Reading Guide.** Although many people prefer to skip prefaces, I suggest that the reader, if he or she is one of them, make an exception for once and read at least this section and the next. The short introductions at the beginning of each chapter are also recommended because they give a summary of the chapter's content.

*Chapter 1* starts from Newton's equations and develops the elementary dynamics of one-, two-, and many-body systems for unconstrained systems. This is the basic material that could be the subject of an introductory course on theoretical physics or could serve as a text for an integrated (experimental and theoretical) course.

*Chapter 2* is the "classical" part of general mechanics describing the principles of canonical mechanics following Euler, Lagrange, Hamilton, and Jacobi. Most of the material is a MUST. Nevertheless, the sections on the symplectic structure of mechanics (Sect. 2.28) and on perturbation theory (Sects. 2.38–40) may be skipped on a first reading.

*Chapter 3* describes a particularly beautiful application of classical mechanics: the theory of spinning tops. The rigid body provides an important and highly nontrivial example of a motion manifold that is not a simple Euclidean space $\mathbb{R}^{2f}$, where $f$ is the number of degrees of freedom. Its rotational part is the manifold of SO(3), the rotation group in three real dimensions. Thus, the rigid body illustrates a Lie group of great importance in physics within a framework that is simple and transparent.

*Chapter 4* deals with relativistic kinematics and dynamics of pointlike objects and develops the elements of special relativity. This may be the most difficult part of the book, as far as the physics is concerned, and one may wish to return to it when studying electrodynamics.

*Chapter 5* is the most challenging in terms of the mathematics. It develops the basic tools of differential geometry that are needed to formulate mechanics in this setting. Mechanics is then described in geometrical terms and its underlying structure is worked out. This chapter is conceived such that it may help to bridge the gap between the more "physical" texts on mechanics and the modern mathematical literature on this subject. Although it may be skipped on a first reading, the tools and the language developed here are essential if one wishes to follow the modern literature on qualitative dynamics.

*Chapter 6* provides an introduction to one of the most fascinating recent developments of classical dynamics: stability and deterministic chaos. It defines and illustrates all important concepts that are needed to understand the onset of chaotic motion and the quantitative analysis of unordered motions. It culminates in a few examples of chaotic motion in celestial mechanics.

*Chapter 7*, finally, gives a short introduction to continuous systems, i.e. systems with an infinite number of degrees of freedom.

*Exercises and Practical Examples.* In addition to the exercises that follow Chaps. 1–6, the book contains a number of practical examples in the form of exercises followed by complete solutions. Most of these are meant to be worked out on a personal computer, thereby widening the range of problems that can be solved with elementary means, beyond the analytically integrable ones. I have tried to choose examples simple enough that they can be made to work even on a programmable pocket computer and in a spirit, I hope, that will keep the reader from getting lost in the labyrinth of computional games.

**Length of this Book.** Clearly there is much more material here than can be covered in one semester. The book is designed for a two-semester course (i.e., typically, an introductory course followed by a course on general mechanics). Even then, a certain choice of topics will have to be made. However, the text is sufficiently self-contained that it may be useful for complementary reading and individual study.

**Mathematical Prerequisites.** A physicist must acquire a certain flexibility in the use of mathematics. On the one hand, it is impossible to carry out all steps in a deduction or a proof, since otherwise one will not get very far with the physics one wishes to study. On the other hand, it is indispensable to know analysis and linear algebra in some depth, so as to be able to fill in the missing links in a logical deduction. Like many other branches of physics, mechanics makes use of many and various disciplines of mathematics, and one cannot expect to have all the tools ready before beginning its study. In this book I adopt the following, somewhat generous attitude towards mathematics. In many places, the details are worked out to a large extent; in others I refer to well-known material of linear algebra and analysis. In some cases the reader might have to return to a good text in mathematics or else, ideally, derive certain results for him- or herself. In this connection it might also be helpful to consult the appendix at the end of the book.

**General Comments and Acknowledgements.** This second English edition follows closely the fifth, enlarged, German edition. As compared to the second English version published in 1994 I have included a more detailed discussion of the tippe top in Chap. 3. Also, Chap. 5 contains an example for the use of Riemannian geometry in mechanics. The book contains the solutions to all exercises, as well as some historical notes on scientists who made important contributions to mechanics and to the mathematics on which it rests.

This book was inspired by a two-semester course on general mechanics that I have taught on and off over the last twenty years at the Johannes Gutenberg University at Mainz and by seminars on geometrical aspects of mechanics. I thank my collaborators, colleagues, and students for stimulating questions, helpful remarks, and profitable discussions. I was happy to realize that the German original, since its first appearance in October 1988, has become a standard text at German speaking universities and I can only hope that it will continue to be equally successful in its English version. I am grateful for the many encouraging reactions and suggestions I have received over the last couple of years. Among those to whom I owe special gratitude are P. Hagedorn, K. Hepp, D. Kastler, H. Leutwyler, N. Papadopoulos, J.M. Richard, G. Schuster, J. Smith, M. Stingl, N. Straumann, W. Thirring, E. Vogt, and V. Vento. Special thanks are due to my former student R. Schöpf who collaborated on the earlier version of the solutions to the exercises. I thank J. Wisdom for his kind permission to use four of his figures illustrating chaotic motions in the solar system, and P. Beckmann who provided

the impressive illustrations for the logistic equation and who advised me on what to say about them.

The excellent cooperation with the team of Springer-Verlag is gratefully acknowledged. Last but not least, I owe special thanks to Dörte for her patience and encouragement.

As with the German edition, I dedicate this book to all those students who wish to study mechanics at some depth. If it helps to make them aware of the fascination of this beautiful field and of physics in general then one of my goals in writing this book is reached.

Mainz, February 1999 *Florian Scheck*

---

I will keep track of possible errata on a page attached to my home page. The latter can be accessed via http://www.thep.physik.uni-mainz.de/staff.html.

# Contents