

CONTENTS

List of Symbols	viii
Preface	ix
1 Existence of Solutions	1
1-1 Introduction	1
1-2 Equations Without Solutions	3
1-3 Integration by Parts	7
1-4 A Necessary Condition	10
1-5 Some Notions From Hilbert Space	11
1-6 Weak Solutions	22
1-7 Operators With Constant Coefficients	24
Problems	27
2 Regularity (Constant Coefficients)	28
2-1 A Necessary Condition	28
2-2 The Friedrichs Mollifier	31
2-3 A Family of Norms	33
2-4 Elliptic Operators	37
2-5 Fourier Transforms	40
2-6 Hypoelliptic Operators	45
2-7 Comparison of Operators	46
2-8 Proof of Regularity	49
2-9 The Closed Graph Theorem	51
Problems	53

3	Regularity (Variable Coefficients)	55
3-1	Formally Hypoelliptic Operators	55
3-2	Proof of Regularity	57
3-3	Vector Spaces	62
3-4	Proof of the Lemmas	65
3-5	Existence	69
3-6	Examples	71
	Problems	71
4	The Cauchy Problem	72
4-1	Statement of the Problem.	72
4-2	Weak Solutions	73
4-3	Hyperbolic Equations	76
4-4	Properties of Hyperbolic Operators	80
4-5	Ordinary Differential Equations	87
4-6	Existence of Solutions	90
4-7	Uniqueness	94
	Problems	98
5	Properties of Solutions	99
5-1	Existence of Strong Solutions	99
5-2	Properties of Strong Solutions	102
5-3	Estimates in One Dimension	104
5-4	Estimates in $n + 1$ Dimensions	111
5-5	Existence Theorems	114
5-6	Properly Hyperbolic Operators	119
5-7	Examples	120
	Problems	121
6	Boundary Value Problems in a Half-Space (Elliptic)	123
6-1	Introduction	123
6-2	The Problem in a Half-Line	124
6-3	Uniqueness	128
6-4	General Boundary Conditions	130
6-5	Estimates for a Simple Case	132
6-6	Estimates for the General Case	136
6-7	Estimates in a Half-Space	139
6-8	Existence in a Half-Space	147
6-9	Some Observations	150
	Problems	151
7	Boundary Value Problems in a Half-Space (Non-Elliptic)	152
7-1	Introduction	152
7-2	The Estimate in a Half-Line	152
7-3	Proof of Theorem 7-1	156

7-4	Hermitian Operators and Matrices	159
7-5	Proof of the Lemmas	163
7-6	Existence and Estimates in a Half-Space	165
7-7	Examples	168
7-8	Nonvanishing Boundary Conditions	171
	Problems	174
8	The Dirichlet Problem	175
8-1	Introduction	176
8-2	A Weak Solution	176
8-3	Normal Boundary Operators	178
8-4	The Estimate	181
8-5	Compact Operators	185
8-6	Compact Embedding	186
8-7	Solving the Problem	193
8-8	Some Theorems in Half-Space	194
8-9	Regularity at the Boundary	198
	Problems	201
9	General Domains	203
9-1	The Basic Theorem	203
9-2	An Inequality and a Regularity Theorem	205
9-3	Localization	209
9-4	Some Lemmas	211
9-5	The Inequality	212
9-6	Strongly Elliptic Operators	214
9-7	Gårding's Inequality	216
9-8	Strong and Weak Solutions	218
9-9	The Exceptional Set	219
	Problems	221
10	General Boundary Value Problems	223
10-1	Formulation of the Problem	223
10-2	The Problem in σ_R	225
10-3	The Solution	228
10-4	The Adjoint System	230
10-5	The Regularity Theorem	233
10-6	The Inequality	235
10-7	The Global Adjoint Operators	235
10-8	The Boundary Norm	237
10-9	The Compactness Argument	239
	Problems	241
	Bibliography	243
	Subject Index	245