Undergraduate Texts in Mathematics

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Winfried Scharlau
Hans Opolka

From Fermat to Minkowski

Lectures on the Theory of Numbers and Its Historical Development

With 31 Illustrations

Springer Science+Business Media, LLC

Winfried Scharlau
Universität Münster
Mathematisches Institut
Fachbereich 15 Mathematik
4400 Münster
West Germany

Hans Opolka
Universität Münster
Mathematisches Institut
Fachbereich 15 Mathematik
4400 Münster
West Germany

Translators

Walter K. Bühler
Springer-Verlag
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U.S.A.

Gary Cornell
Department of Mathematics
University of Connecticut
Storrs, CT 06268
U.S.A.

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Preface

This book arose from a course of lectures given by the first author during the winter term 1977/1978 at the University of Münster (West Germany). The course was primarily addressed to future high school teachers of mathematics; it was not meant as a systematic introduction to number theory but rather as a historically motivated invitation to the subject, designed to interest the audience in number-theoretical questions and developments. This is also the objective of this book, which is certainly not meant to replace any of the existing excellent texts in number theory. Our selection of topics and examples tries to show how, in the historical development, the investigation of obvious or natural questions has led to more and more comprehensive and profound theories, how again and again, surprising connections between seemingly unrelated problems were discovered, and how the introduction of new methods and concepts led to the solution of hitherto unassailable questions. All this means that we do not present the student with polished proofs (which in turn are the fruit of a long historical development); rather, we try to show how these theorems are the necessary consequences of natural questions.

Two examples might illustrate our objectives. The book will be successful if the reader understands that the representation of natural numbers by quadratic forms—say, $n = x^2 + dy^2$—necessarily leads to quadratic reciprocity, or that Dirichlet, in his proof of the theorem on primes in arithmetical progression, simply had to find the analytical class number formula. This is why, despite some doubts, we retained the relatively amorphous, unsystematic and occasionally uneconomical structure of the original lectures in the book. A systematic presentation, with formal definitions, theorems, proofs and remarks would not have suited the real purpose of this course, the description of living developments. We nevertheless hope that the reader, with the occasional help of a supplementary text, will be able to learn a number of subjects from this book such as the theory of binary quadratic forms or of continued fractions or important facts on L-series and ζ-functions.

Clearly, we are primarily interested in number theory but we present it not as a streamlined ready-made theory but in its historical genesis, however, without inordinately many detours. We also believe that the lives and times of the mathematicians whose works we study are of intrinsic interest; to learn something about the lives of Euler and Gauss is a sensible supplement to learning mathematics. What was said above also applies to the history in this book: we do not aim at completeness but hope to stir up the interests of our readers by confining ourselves to a few themes and hope this will give enough motivation to study some of the literature quoted in our text.

Many persons have contributed to this book. First of all, the students of the course showed a lot of enthusiasm for the subject and made it worthwhile to prepare a set of notes; Walter K. Bühler kindly suggested to publish these notes in book form and prepared the English translation. Gary Cornell helped with the translation and suggested several mathematical improvements; many colleagues and friends contributed encouragement and mathematical and historical comments and pointed out a number of embarrassing errors. We wish to mention in particular Harold Edwards, Wulf-Dieter Geyer, Martin Kneser, and Olaf Neumann. It is a pleasure to thank them all.

Münster, West Germany
June 1984

WINFRIED SCHARLAU
HANS OPOLKA

Added in proof. In early 1984, André Weil's Number Theory: *An Approach Through History from Hammurapi to Legendre* appeared. It contains substantial additional material and discussion, especially concerning the period between Fermat and Legendre.

Contents

Literature

More references are given at the end of the individual chapters.

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