

Friedrich Sauvigny

# Partial Differential Equations 2

Functional Analytic Methods

With Consideration of Lectures  
by E. Heinz

 Springer

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