Applied Mathematical Sciences

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Semidynamical Systems in Infinite Dimensional Spaces

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To my parents

PREFACE

Where do solutions go, and how do they behave en route? These are two of the major questions addressed by the qualitative theory of differential equations. The purpose of this book is to answer these questions for certain classes of equations by recourse to the framework of semidynamical systems (or topological dynamics as it is sometimes called). This approach makes it possible to treat a seemingly broad range of equations from nonautonomous ordinary differential equations and partial differential equations to stochastic differential equations. The methods are not limited to the examples presented here, though.

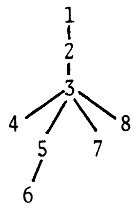
The basic idea is this: Embed some representation of the solutions of the equation (and perhaps the equation itself) in an appropriate function space. This space serves as the phase space for the semidynamical system. The phase map must be chosen so as to generate solutions to the equation from an initial value. In most instances it is necessary to provide a "weak" topology on the phase space. Typically the space is infinite dimensional.

These considerations motivate the requirement to study semidynamical systems in non locally compact spaces. Our objective here is to present only those results needed for the kinds of applications one is likely to encounter in differential equations. Additional properties and extensions of abstract semidynamical systems are left as exercises. The power of the semidynamical framework makes it possible to characterize the asymptotic behavior of the solutions of such a wide class of equations.

A caveat is in order. The stability results obtained in many of the examples can be gotten directly without recourse to the abstract semidynamical system setting. Moreover, in some instances, sharper results can be obtained by utilizing special techniques and methods suitably adjusted to that particular equation. On the other hand, the generality of the semidynamical system approach allows for a greater understanding of the unifying concepts running through all of the examples.

The first three chapters are devoted to the theory of semidynamical systems. Virtually all of the results hold for a discrete time parameter as well as a continuous time parameter. Because of their simplicity some examples of discrete semidynamical systems are included to illustrate the variety of asymptotic behavior. The remainder of the book is devoted to applications of the theory. The range of applications reflects recent mathematical activity. The choice of examples, though, reflects my interests and bias as well.

The presentation is meant to be self contained (except for a few lapses in Chapters 4, 5, and 7, where references are supplied). Appendices on functional analysis and probability are provided for this purpose. Definitions of terms not found in the text can usually be found in one of the appendices. Each chapter concludes with a set of exercises and a section called "Notes and Comments." This provides the reader with the source of the results of that chapter. It also offers some commentary and related results. Most of the source material is from the late 1960's and 1970's. The reader should be familiar with real analysis on the level of Royden [1] and ordinary differential equations on the level of Hirsch and Smale [1]. A little knowledge of partial differential equations in Chapter 5 and Markov processes in Chapter 7 would be useful. The chapter dependence is as follows:



I want to acknowledge the contributions of many people. The initial impetus for this book came from Nam Bhatia. Much of the first chapter is based on his notes. Jim Yorke and Wei Shaw read portions of the manuscript. I am grateful for their helpful comments. Numerous colleagues have assisted me through their participation in seminars based on this material. Marshall Slemrod read the entire manuscript and provided invaluable suggestions which I feel improve the manuscript. A number of reviewers and referees also provided helpful criticisms and suggestions for improvement. Any remaining faults are mine.

The following people typed portions of the manuscript at various stages of its evolution: Pam Lambert, Mary Beth Minton, Nancy Dame, Carol Granis, and Susie Evers. Kate MacDougall typed the final camera-ready copy. I am grateful for their careful work and seemingly unbounded patience. Thanks are also due to Leon Booth, former Dean of CAS, for financial support in the preparation of the manuscript.

Fairfax, Virginia
August 1981

Stephen Saperstone

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