Muhammad Sahimi

Heterogeneous Materials I Linear Transport and Optical Properties

With 145 Illustrations

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