TRANSLATIONS OF
MATHEMATICAL
MONOGRAPHS
VOLUME 155

Yu. Safarov
D. Vassiliev

The Asymptotic Distribution of Eigenvalues of Partial Differential Operators

American Mathematical Society

Contents

Preface xi

Chapter 1. Main Results 1
1.1. Statement of the spectral problem 1
1.2. One-term asymptotic formula for $N(\lambda)$ 9
1.3. Hamiltonian billiards I: basic definitions and results 11
1.4. Hamiltonian billiards II: reflection matrix 27
1.5. Hamiltonian billiards III: Maslov index 32
1.6. Classical two-term asymptotic formula for $N(\lambda)$ 37
1.7. Nonclassical two-term asymptotic formulae for $N(\lambda)$ 49
1.8. Two-term asymptotic formulae for the spectral function 56

Chapter 2. Oscillatory Integrals 63
2.1. Local oscillatory integrals and pseudodifferential operators 63
2.2. Global oscillatory integrals 70
2.3. Homogeneous canonical transformations 74
2.4. Phase functions associated with homogeneous canonical transformations 78
2.5. Restriction of phase functions to the boundary 89
2.6. Extension of phase functions from the boundary 91
2.7. Standard oscillatory integrals associated with homogeneous canonical transformations 100
2.8. Boundary layer oscillatory integrals associated with homogeneous canonical transformations 112
2.9. Boundary oscillatory integrals associated with homogeneous canonical transformations 115
2.10. Parameter-dependent oscillatory integrals 119

Chapter 3. Construction of the Wave Group 129
3.1. Characteristic properties of distributions associated with the wave group 130
3.2. Representation of the wave group by means of oscillatory integrals: sufficient conditions 140
3.3. Representation of the wave group by means of oscillatory integrals: effective construction for manifolds without boundary 142
3.4. Representation of the wave group by means of oscillatory integrals: effective construction for manifolds with boundary 148
3.5. Construction of the wave group when the source is close to the boundary 161

Chapter 4. Singularities of the Wave Group 167
4.1. Singularities of Lagrangian distributions 168
4.2. Singularity of the wave group at $t = 0$ 176
4.3. Singularities of the wave group at $t \neq 0$ for admissible pseudodifferential cut-offs 179
4.4. Singularities of the wave group at $t \neq 0$ for nonadmissible pseudodifferential cut-offs 183
4.5. Singularity of the wave group at $t = 0$ when the source is close to the boundary 190

Chapter 5. Proof of Main Results 193
5.1. Partition of the manifold M into three zones 193
5.2. Asymptotics of the trace of the spectral projection in the interior zone 198
5.3. Asymptotics of the trace of the spectral projection in the intermediate zone 201
5.4. Asymptotics of the trace of the spectral projection in the boundary zone 205
5.5. Asymptotics of the spectral function 224

Chapter 6. Mechanical Applications 229
6.1. Membranes and acoustic resonators 229
6.2. Elastic plates 230
6.3. Two- and three-dimensional elasticity 236
6.4. Elastic shells 239
6.5. Hydroelasticity 247

Appendix A. Spectral Problem on the Half-line
by A. HOLST 251
A.1. Basic facts 251
A.2. The reflection matrix 263
A.3. Trace formulae 276
A.4. Dependence on parameters 292

Appendix B. Fourier Tauberian Theorems
by M. LEVITIN 297
B.1. Introductory remarks 297
B.2. Basic theorem 298
B.3. Rough estimate for the nonzero singularities 300
B.4. General refined theorem 301
B.5. Special version of the general refined theorem 304

Appendix C. Stationary Phase Formula 307

Appendix D. Hamiltonian Billiards: Proofs 313
D.1. Measure of "awkward" starting points 313
D.2. Dead-end trajectories 318
D.3. Convexity and concavity 321
D.4. Measurability of sets and functions 324
D.5. Lengths of loops and periodic trajectories 326
D.6. Maslov index 328

Appendix E. Factorization of Smooth Functions and Taylor-type Formulae 335

References 343

Principal Notation 349

Index 353