Universitext

Joseph Rotman

Galois Theory

Springer-Verlag
New York Berlin Heidelberg
London Paris Tokyo Hong Kong

Joseph Rotman
Department of Mathematics
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801
USA

Mathematics Subject Classification (1980): 12-01, 12F10

Library of Congress Cataloging-in-Publication Data
Rotman, Joseph J.
Galois theory / Joseph Rotman.
p. cm.—(Universitext)
Includes bibliographical references.
1. Galois theory. I. Title.
QA171.R668 1990
512'.3—dc20 90-9740

Printed on acid-free paper.

Camera-ready copy prepared using LaTEX.

9 8 7 6 5 4 3 2 1

ISBN-13: 978-0-387-97305-0 e-ISBN-13: 978-1-4684-0367-1
DOI: 10.1007/978-1-4684-0367-1

To my teacher Irving Kaplansky

Preface

This little book is designed to teach the basic results of Galois theory—fundamental theorem; insolvability of the quintic; characterization of polynomials solvable by radicals; applications; Galois groups of polynomials of low degree—efficiently and lucidly. It is assumed that the reader has had introductory courses in linear algebra (the idea of the dimension of a vector space over an arbitrary field of scalars should be familiar) and "abstract algebra" (that is, a first course which mentions rings, groups, and homomorphisms). In spite of this, a discussion of commutative rings, starting from the definition, begins the text. This account is written in the spirit of a review of things past, and so, even though it is complete, it may be too rapid for one who has not seen any of it before. The high number of exercises accompanying this material permits a quicker exposition of it. When I teach this course, I usually begin with a leisurely account of group theory, also from the definition, which includes some theorems and examples that are not needed for this text. Here I have decided to relegate needed results of group theory to appendices: a glossary of terms; proofs of theorems. I have chosen this organization of the text to emphasize the fact that polynomials and fields are the natural setting, and that groups are called in to help.

A thorough discussion of field theory would have delayed the journey to Galois's Great Theorem. Therefore, some important topics receive only a passing nod (separability, cyclotomic polynomials, norms, infinite extensions, symmetric functions) and some are snubbed altogether (algebraic closure, transcendence degree, resultants, traces, normal bases, Kummer theory). My belief is that these subjects should be pursued only after the reader has digested the basics.

My favorite expositions of Galois theory are those of E. Artin, Kaplansky, and van der Waerden, and I owe much to them. For the appendix on "old-fashioned Galois theory," I relied on recent accounts, especially [Edwards], [Gaal], [Tignol], and [van der Waerden, 1985], and older books, especially [Dehn] (and [Burnside and Panton], [Dickson], and [Netto]). I thank my colleagues at the University of Illinois, Urbana, who, over the years, have clarified obscurities; I also thank Peter Braunfeld for suggestions that im-

proved Appendix 3 and Peter M. Neumann for his learned comments on Appendix 4.

I hope that this monograph will make both the learning and the teaching of Galois theory enjoyable, and that others will be as taken by its beauty as I am.

Joseph Rotman
Urbana, Illinois, 1990

To the Reader

Regard the exercises as part of the text; read their statements and do attempt to solve them. A star before an exercise indicates that it will be mentioned elsewhere in the text, perhaps in a proof. A result labeled Theorem 5 is the fifth theorem in the text; Theorem A5 is the fifth theorem in Appendix 2 (group theory); Theorem B5 is the fifth theorem in Appendix 3 (ruler-compass constructions).

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