

Clay C. Ross

Differential Equations

An Introduction with Mathematica®

Second Edition

With 88 Figures

 Springer

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