

Derek J.S. Robinson

A Course in the Theory of Groups

Second Edition

With 40 Illustrations



Springer

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