



# **MATHEMATICAL LOGIC**

*A First Course*

# CONTENTS

<b>PREFACE</b>	v
<b>NOTE TO THE STUDENT</b>	xi
<b>Chapter 1 The Propositional Calculus</b>	1
§1 Formation Rules for P	4
§2 Formal Semantics of P	8
§3 Axiomatization of P	14
§4 The Deduction Theorem	16
§5 Some Theorem Schemas of P	20
§6 Completeness	22
<b>Chapter 2 First-Order Logic</b>	32
§7 Formation Rules for $L(\mathfrak{S}, X)$	34
§8 Free and Bound Variables	37
§9 Models	38
§10 Validity	41
§11 Axiomatization of $L(\mathfrak{S}, X)$	43
§12 Propositional Calculus	44
§13 The Deduction Theorem	45

§14	Some Theorems of First-Order Logic	48
§15	The Soundness Theorem	50
§16	The Extension Lemma	52
§17	The Completeness Theorem	54
§18	First-Order Logic with Equality	56
 <b>Chapter 3 First-Order Recursive Arithmetic</b>		<b>66</b>
§19	Primitive Recursive Functions	66
§20	Some Primitive Recursive Functions	69
§21	Formation Rules for RA	72
§22	Semantics	76
§23	Axiomatics	78
§24	Soundness	80
§25	Sentences without Variables	81
 <b>Chapter 4 Arithmetization of Syntax</b>		<b>90</b>
§26	Finite Sums and Products	90
§27	Primitive Recursive Relations	91
§28	Definition by Cases and the Least Number Operator	94
§29	Prime Numbers	96
§30	Primitive Recursiveness of the Prime Decomposition	99
§31	Gödel Numbers	100
§32	Primitive Recursiveness of the Notion of Function Constant	103
§33	Primitive Recursiveness of the Notion of Term	105
§34	Primitive Recursiveness of the Notion of Wff	106
§35	Substitution	107
§36	Deductions	109

<b>Chapter 5</b>	<b>The Incompleteness Theorems and Other Applications of the Liar Paradox</b>	111
§37	The First Gödel Incompleteness Theorem	112
§38	Proof of the First Gödel Theorem in RA	116
§39	The Second Gödel Incompleteness Theorem	117
§40	Tarski's Theorem	119
§41	Arithmetical Sets	121
§42	Recursive Sets and Decision Procedures	122
§43	Church's Theorem for RA	126
<b>Chapter 6</b>	<b>Second-Order Logic</b>	132
§44	Formation Rules for $L_2(\mathfrak{S}, X)$	133
§45	Axiomatics for $L_2(\mathfrak{S}, X)$	134
§46	Semantics of Second-Order Logic	137
§47	The Completeness Theorem for Second-Order Logic	140
§48	Equality	142
§49	Second-Order Peano Arithmetic	145
§50	Primitive Recursive Functions	148
§51	Numeralwise Representability	153
§52	Gödel's Theorem for $PA_2$	156
§53	Church's Theorem for Second-Order Logic	158
§54	Categoricity of $PA_2$	161
§55	Skolem's Paradox	164
§56	Formal Set Theory	166
<b>Appendix</b>		171
§A1	Sets and Functions	171
§A2	Induction	175
§A3	Primitive Recursion	178
§A4	Countable Sets	180

ANSWERS TO SELECTED EXERCISES	183
SUGGESTED READING	199
INDEX OF SYMBOLS	203
INDEX	209