

Fundamentals of Cosmology

Physics and Astronomy  **ONLINE LIBRARY**
<http://www.springer.de/phys/>

Springer-Verlag Berlin Heidelberg GmbH



POLYTECHNIQUE

The Ecole Polytechnique, one of France's top academic institutions, has a long-standing tradition of producing exceptional scientific textbooks for its students. The original lecture notes, the *Cours de l'Ecole Polytechnique*, which were written by Cauchy and Jordan in the nineteenth century, are considered to be landmarks in the development of mathematics.

The present series of textbooks is remarkable in that the texts incorporate the most recent scientific advances in courses designed to provide undergraduate students with the foundations of a scientific discipline. An outstanding level of quality is achieved in each of the seven scientific fields taught at the *Ecole*: pure and applied mathematics, mechanics, physics, chemistry, biology, and economics. The uniform level of excellence is the result of the unique selection of academic staff there which includes, in addition to the best researchers in its own renowned laboratories, a large number of world-famous scientists, appointed as part-time professors or associate professors, who work in the most advanced research centers France has in each field.

Another distinctive characteristic of these courses is their overall consistency; each course makes appropriate use of relevant concepts introduced in the other textbooks. This is because each student at the Ecole Polytechnique has to acquire basic knowledge in the seven scientific fields taught there, so a substantial link between departments is necessary. The distribution of these courses used to be restricted to the 900 students at the Ecole. Some years ago we were very successful in making these courses available to a larger French-reading audience. We now build on this success by making these textbooks also available in English.

James Rich

Fundamentals of Cosmology

With 98 Figures, 12 Tables,
and 68 Problems with 23 Selected Solutions
Solutions Manual for Instructors on Request
Directly from Springer-Verlag



Springer

Professor James Rich

CEA-Saclay

DAPNIA/SPP

91191 Gif-sur-Yvette, France

E-mail: rich@hep.saclay.cea.fr

Cover picture: The front cover shows Figs. 5.3 and 7.10.

Library of Congress Cataloging-in-Publication Data. Rich, James, 1952– Fundamentals of cosmology/
James Rich. p. cm. Includes bibliographical references and index.

ISBN 978-3-642-07461-5 ISBN 978-3-662-04446-9 (eBook)

DOI 10.1007/978-3-662-04446-9

Cosmology. I. Title. QB981.R486 2001 523.1–dc21 2001020334

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

<http://www.springer.de>

© Springer-Verlag Berlin Heidelberg 2001

Originally published by Springer-Verlag Berlin Heidelberg New York in 2001

Softcover reprint of the hardcover 1st edition 2001

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Data prepared by the author using a Springer \TeX macro package

Cover design: *design & production* GmbH, Heidelberg

Printed on acid-free paper SPIN: 10789949 56/3141/ba - 5 4 3 2 1 0

Preface

This is a textbook intended for students and researchers who wish to understand the physics of standard “big bang” cosmology and how it is used to interpret the most recent observations. It is based on courses given over the last seven years to beginning graduate students at the University of Paris and to advanced undergraduates at l’Ecole Polytechnique. Since the great majority of these students did not intend to become professional cosmologists, I have emphasized subjects that should be of general interest.

Progress in observations over the last ten years has been truly astounding and a new textbook might be justified simply to report on recent breakthroughs. The traditional successes of modern cosmology are well-known. Among these are the dynamical understanding of the universal expansion, the prediction of the cosmic microwave background radiation, and the calculation of the abundances of the light elements. To these we can add new observations that suggest that we are beginning the era of “precision cosmology.” Perhaps most spectacular was the observation this year of the first acoustic peak in the anisotropy spectrum of the cosmic background radiation by the Boomerang and Maxima collaborations. These beautiful measurements have convinced many people that the universe has a nearly critical energy density and that a complete understanding of structure formation may be at hand.

While a critical density was expected by many cosmologists, the observed breakdown into different components has revolutionary implications. Observations during the last decade have confirmed that most of the matter that is bound in galaxies or galaxy clusters is in some unknown form. Many cosmologists believe that the observations indicate the existence of “cold dark matter”, most likely some as yet undetected weakly interacting massive particle. Cold dark matter has been a standard fixture on the conference circuit for nearly twenty years, and we sometimes forget how daring this prediction is.

More revolutionary still is the conclusion, based on the observed fluxes from high-redshift supernovae, that the expansion of the universe is accelerating. Within standard gravitational theory, this implies that the energy content of the universe is dominated by an effective vacuum energy or, equivalently, a cosmological constant. Being a new form of energy not directly as-

sociated with an elementary particle, this discovery, if confirmed, would rank in theoretical importance with the discovery of, say, electromagnetic fields.

Observations during the next decade will provide precision tests of this picture of a universe dominated by cold dark matter and vacuum energy. A more difficult problem is to determine whether these two substances are “elements of reality” or just elements of theories. Even if the Universe *acts* like a universe governed by general relativity with a mixture of cold dark matter and vacuum energy, it is possible that nature has fooled us because of our ignorance of a key ingredient. For example, a model using only ordinary matter but with some sort of “modified gravity” operating at cosmological scales might also agree with observations. Some have argued that this is suggested by the fact that models using the simplest cold dark matter particles apparently do not accurately predict the structure of galactic cores or the number of small galaxies. Time will tell if these objections to the standard model hold up. If they do, things will be quite confusing if we have to rely on cosmological observations to *determine* the correct laws of gravity. It would be better if someone settles the question by directly detecting the dark-matter particles.

Given the fascinating questions addressed by cosmology and the great interest aroused by vigorous observational programs, it is not surprising that many students wish to study the subject before completely mastering the necessary background from observational astronomy and astrophysics, elementary particle physics, nuclear physics, and general relativity. This book is an attempt to address this problem.

General relativity is certainly the most difficult aspect of cosmological theory and it presents a formidable pedagogical challenge for an introductory course. Originally, I used the usual Newtonian derivations of the Friedmann equation but this is ultimately unsatisfying. Finally, I have adopted the strategy of presenting a self-contained introduction to relativistic gravitation that uses only the mathematics that is absolutely necessary for cosmology. This is possible because of the extreme simplicity of homogeneous cosmology. We will obtain all the results we need without mentioning affine connections or covariant derivatives.

While attempting to be “relativistically correct”, I have adopted a strictly phenomenological point of view of general relativity where the mathematics never strays far from observations made with clocks and radar ranging devices. For instance, comoving coordinates are defined operationally before finding the Robertson–Walker metric from general considerations of symmetry. This strategy is meant to attack what appears to be one of the greatest difficulties of general relativity, connecting all those symbols with the measurements.

In the same phenomenological spirit, in one chapter we abandon the usual comoving coordinates and adopt a simple system that can be constructed operationally by one freely falling observer. In such coordinates, the metric is

locally Lorentzian, and many things that are mysterious in comoving coordinates become relatively clear. In particular, it is easy to derive the Friedmann equation, and the nature of the mysterious vacuum energy is made at least plausible.

Concerning elementary particle physics and nuclear physics, I have mostly taken the point of view that these disciplines exist simply to furnish cosmologists with a list of known and hypothetical particles and the values of their cross-sections. Hence, I have not attempted any detailed theoretical introduction to these two fascinating subjects. Speculative subjects like supersymmetric dark matter and inflationary and quintessential scalar fields are treated phenomenologically with only brief mention of the difficulties encountered in integrating them into a coherent theory of particle physics.

Finally, concerning astronomy and astrophysics, I have tried to provide the minimum background necessary to understand the observations. Measurements are often presented in relatively undigested forms so that students can get a feeling for the quality of the data and the difficulty in analyzing it. The importance of hypotheses used in the interpretation of the often ambiguous astrophysical data is emphasized.

I have not gone upstream of the data to discuss observing techniques. This means that I have not presented in the detail it deserves the important technological advances that have made the observations possible. Among these advances we can mention the new generation of 10-m-class telescopes and the Hubble Space Telescope that have given us a much clearer visual view of distant objects. Space-based X-ray telescopes have permitted the detailed study of galaxy clusters, the largest bound objects in the universe. All these telescopes have generated enormous amounts of high-quality data because of advances in photon detection technology. Most obvious are the new CCDs that have gradually replaced traditional photographic plates. Large CCD mosaics have permitted the discovery of high-redshift supernovae, the completion of enormous redshift surveys, and the mapping of mass distributions through weak gravitational lensing. We mention also the new cryogenic bolometers that were used in the measurements of Boomerang and Maxima, and that may someday allow the detection of dark-matter particles.

It has also not been possible to discuss the techniques of computer simulations that are so important for the understanding of structure formation. Our discussion of this process will be, therefore, quite qualitative. We do not touch the unsolved problem of how star-formation is first triggered, creating the observable universe of galaxies. Until astronomers succeed in completely determining the matter distribution of the Universe using gravitational lensing, this problem will continue to plague structure studies based on counting visible galaxies.

Finally, I have not reviewed the history of modern cosmology. This story starts with the discovery of the universal expansion by Hubble and its interpretation by Lemaitre. It is followed by Gamow's theory of primordial

nucleosynthesis and the prediction of the cosmic background radiation and the confirming observations of Penzias and Wilson. This story is, by now, well-known so I have mostly ignored it. As a result, references to pioneering work have been perhaps neglected in favor of the most recent work.

Many people have made contributions to this work. Most important are my students at the DEA de Champs, Matière et Particules and the DEA de Physique Théorique. The questions that they asked and the questions that I thought they might ask have constantly challenged me. Special thanks to the student who glared at me when I told her that if she wanted to know where the Friedmann equation comes from she should take a class in general relativity. Chapters 3 and 4 sprang from that tense moment.

This book would never have become a book without the encouragement and advice of Jean-Louis Basdevant. He also suggested that I try it out on undergraduates, an experience that forced me to clarify much of the basic physics.

The following people have read all or parts of the manuscript and made important suggestions: Alexis Amadon, Jean-Louis Basdevant, Guillaume Blanc, Alain Blanchard, Jean-Francois Glicenstein, David Langlois, Thierry Lasserre, Alain Milsztajn, David Lloyd Owen, Charling Tao, Dominique Yvon, and especially Jacques Haissinski. They found many (though not all) errors and kindly pointed out passages that were not quite clear enough.

I have benefited from discussions with many colleagues on recent research. Special thanks to Monique Arnaud, Christophe Balland, Marc Besancon, Pierre Binétruy, Alain Blanchard, Jim Bartlett, Nathalie Deruelle, Ken Ganga, Andy Gould, David Graff, Michael Joyce, Boris Kayser, David Langlois, Christophe Magneville, Alain Milsztajn, Yannick Mellier, Robert Mochkovitch, Reynald Pain, Marguerite Pierre, Joe Silk, Michel Spiro, Elizabeth Vangioni-Flan, and to all the others whom I have forgotten.

Unpublished figures were contributed by Monique Arnaud, Ken Ganga, Thierry Lasserre, and Elizabeth Flan-Vangioni. Figures from the web were kindly contributed by the European Southern Observatory, NASA and the COBE working group, The European Space Agency, and the Supernova Cosmology Project. Thanks to Albert Bosma for resurrecting Fig. 2.25. Nathalie Palanque Delabrouille was the one who succeeded in getting them all to print out. The people at Springer, especially Jacqueline Lenz, H.J. Kölsch and Claus-Dieter Bachem, were always there to help with technical and aesthetic questions. Finally, thanks to Pascale for installing Linux and for much more.

Gif-sur-Yvette,
January 2001

James Rich

Contents

1. Introduction	1
1.1 The Composition of the Universe	3
1.1.1 The Visible Universe: Galaxies	3
1.1.2 Baryons	10
1.1.3 Cold Dark Matter	11
1.1.4 Photons	12
1.1.5 Neutrinos	13
1.1.6 The Vacuum	15
1.2 The Evolution of the Universe	17
1.2.1 The Scale Factor $a(t)$	17
1.2.2 Gravitation and the Friedmann Equation	19
1.2.3 Open and Closed Universes	21
1.2.4 The Evolution of the Temperature	23
1.2.5 An Improved Friedmann Equation	28
1.2.6 The Evolution of the Ω s and Structure Formation	30
1.2.7 The Standard Scenario	32
1.3 Open Questions	34
Exercises	38
2. Observational Cosmology	39
2.1 Stars and Quasi-stars	39
2.2 Galaxies	51
2.3 Galaxy Clusters	54
2.4 Dark Matter	57
2.4.1 Wimps	59
2.4.2 Axions	62
2.4.3 Baryonic Dark Matter	62
2.5 The Cosmological Parameters	67
2.5.1 H_0	67
2.5.2 ρ s and Ω s	70
Exercises	74

3. Coordinates and Metrics	85
3.1 Relativity and Gravitation	88
3.2 Comoving coordinates	93
3.3 The Metric I: Mostly Isotropy	96
3.4 The Metric II: Mostly Homogeneity	99
3.5 Photon Propagation	103
3.6 The Luminosity and Angular Distances	106
3.7 The Geodesic Equation	108
3.8 Gravitational Lensing	111
Exercises	118
4. The Field Equations	125
4.1 Our Freely Falling Coordinates	126
4.2 The Energy-Momentum Tensor	129
4.3 The Friedmann Equation	134
4.4 The Cosmological Parameters	136
4.5 Scalar Fields	137
4.6 The Riemann Tensor	139
4.7 A Universe with $\rho = 0$	142
4.8 The Einstein Tensor	143
4.9 The General Einstein Equation	144
Exercises	148
5. Friedmannology	151
5.1 The Age of the Universe	153
5.2 Luminosity and Angular Distances	156
5.3 The Horizon Problem	162
5.4 The Ω Problem	168
5.5 Inflation	169
5.6 Intergalactic Scattering and Absorption	172
Exercises	174
6. The Thermal History of the Universe	179
6.1 Equilibrium Distributions	182
6.2 The Boltzmann Equation	186
6.3 Electrons and Positrons	191
6.4 Neutrinos	196
6.5 Primordial Nucleosynthesis	198
6.6 Wimps	206
6.7 Baryogenesis	209
6.8 Irreversibility	210
6.9 The Future	212
Exercises	214

7. Structure Formation 221

7.1 A Spherical Collapse Model 226

 7.1.1 The Metric 227

 7.1.2 Expansion and Collapse 228

 7.1.3 The Linear Regime 232

7.2 The Spectrum of Density Fluctuations 234

7.3 Newtonian Evolution 240

7.4 Hubble Exit and Entry 244

7.5 The Primordial Spectrum 247

7.6 Cold Dark Matter Models 252

7.7 Neutrinos and Baryons 254

7.8 Photon Propagation 256

7.9 CBR Anisotropies 259

 7.9.1 The Sources of Anisotropy 263

 7.9.2 $\Delta\theta > \theta_H$ 264

 7.9.3 $\Delta\theta < \theta_H$ 265

 7.9.4 The Cosmological Parameters 268

Exercises 269

Appendix 275

A Lorentz Vectors and Tensors 275

B Natural Units 277

C Standard Particles and Beyond 279

D Magnitudes 283

E Useful Formulas and Numbers 285

F Solutions and Hints for Selected Exercises 288

References 293

Index 299