

# A MATHEMATICAL VIEW OF INTERIOR-POINT METHODS IN CONVEX OPTIMIZATION

**James Renegar**

Cornell University  
Ithaca, New York

**siam**

Society for Industrial and Applied Mathematics  
Philadelphia

**MPS**

Mathematical Programming Society  
Philadelphia

Copyright ©2001 by the Society for Industrial and Applied Mathematics.

10 9 8 7 6 5 4 3 2 1

All rights reserved. Printed in the United States of America. No part of this book may be reproduced, stored, or transmitted in any manner without the written permission of the publisher. For information, write to the Society for Industrial and Applied Mathematics, 3600 University City Science Center, Philadelphia, PA 19104-2688.

**Library of Congress Cataloging-in-Publication Data**

Renegar, James, 1955-

A mathematical view of interior-point methods in convex optimization / James Renegar.

p. cm. – (MPS-SIAM series on optimization)

Includes bibliographical references and index.

ISBN 0-89871-502-4 (pbk.)

1. Interior-point methods. 2. Mathematical optimization. 3. Convex programming. I. Title. II. Series.

QA402.5 .R46 2001

519.3-dc21

2001042999

This research was supported by NSF grant CCR-9403580.

# Contents

<b>Preface</b>		<b>vii</b>
<b>1</b>	<b>Preliminaries</b>	<b>1</b>
1.1	Linear Algebra . . . . .	2
1.2	Gradients . . . . .	5
1.3	Hessians . . . . .	9
1.4	Convexity . . . . .	11
1.5	Fundamental Theorems of Calculus . . . . .	14
1.6	Newton's Method . . . . .	18
<b>2</b>	<b>Basic Interior-Point Method Theory</b>	<b>21</b>
2.1	Intrinsic Inner Products . . . . .	21
2.2	Self-Concordant Functionals . . . . .	23
	2.2.1 Introduction . . . . .	23
	2.2.2 Self-Concordancy and Newton's Method . . . . .	27
	2.2.3 Other Properties . . . . .	31
2.3	Barrier Functionals . . . . .	35
	2.3.1 Introduction . . . . .	35
	2.3.2 Analytic Centers . . . . .	38
	2.3.3 Optimal Barrier Functionals . . . . .	39
	2.3.4 Other Properties . . . . .	40
	2.3.5 Logarithmic Homogeneity . . . . .	41
2.4	Primal Algorithms . . . . .	43
	2.4.1 Introduction . . . . .	43
	2.4.2 The Barrier Method . . . . .	45
	2.4.3 The Long-Step Barrier Method . . . . .	49
	2.4.4 A Predictor-Corrector Method . . . . .	52
2.5	Matters of Definition . . . . .	54
<b>3</b>	<b>Conic Programming and Duality</b>	<b>65</b>
3.1	Conic Programming . . . . .	65
3.2	Classical Duality Theory . . . . .	68
3.3	The Conjugate Functional . . . . .	75
3.4	Duality of the Central Paths . . . . .	81

3.5	Self-Scaled (or Symmetric) Cones . . . . .	83
3.5.1	Introduction . . . . .	83
3.5.2	An Important Remark on Notation . . . . .	85
3.5.3	Scaling Points . . . . .	86
3.5.4	Gradients and Norms . . . . .	90
3.5.5	A Useful Theorem . . . . .	95
3.6	The Nesterov–Todd Directions . . . . .	97
3.7	Primal-Dual Path-Following Methods . . . . .	102
3.7.1	Measures of Proximity . . . . .	102
3.7.2	An Algorithm . . . . .	104
3.7.3	Another Algorithm . . . . .	106
3.8	A Primal-Dual Potential-Reduction Method . . . . .	108
3.8.1	The Potential Function . . . . .	108
3.8.2	The Algorithm . . . . .	110
3.8.3	The Analysis . . . . .	111

**Bibliography** **115**

**Index** **117**