

METHODS OF MODERN MATHEMATICAL PHYSICS

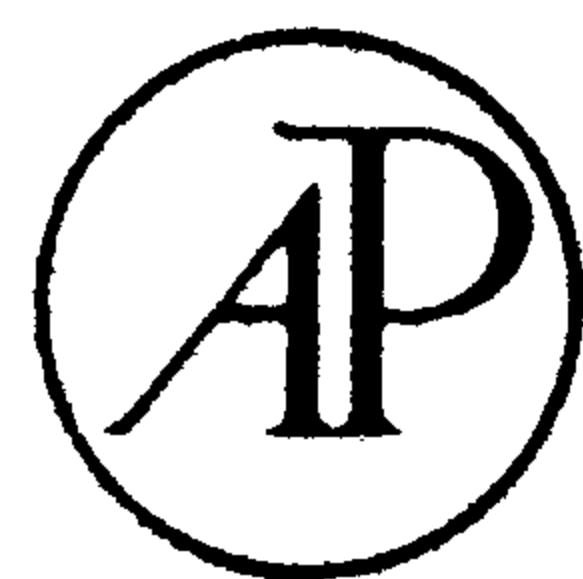
IV: ANALYSIS OF OPERATORS

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