

METHODS OF MODERN MATHEMATICAL PHYSICS

I: FUNCTIONAL ANALYSIS

Revised and Enlarged Edition

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To

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