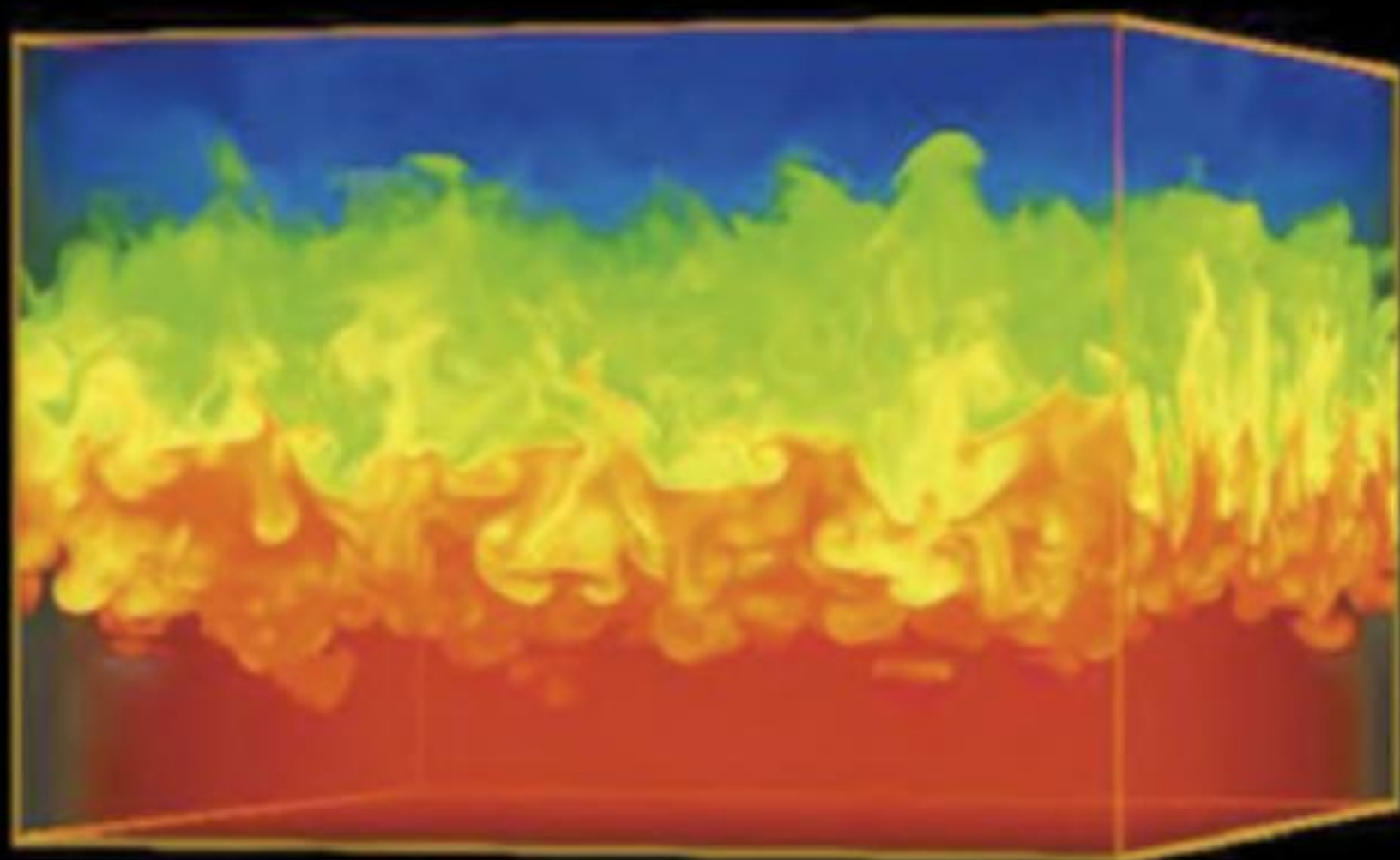


J. N. REDDY



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MECHANICS**

A Study of Conservation Principles with Applications

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