Texts in Applied Mathematics 27

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B. Daya Reddy

Introductory Functional Analysis

With Applications to Boundary Value Problems and Finite Elements

With 145 Illustrations

Springer

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Series Preface

Mathematics is playing an ever more important role in the physical and biological sciences, provoking a blurring of boundaries between scientific disciplines and a resurgence of interest in the modern as well as the classical techniques of applied mathematics. This renewal of interest, both in research and teaching, has led to the establishment of the series: *Texts in Applied Mathematics* (*TAM*).

The development of new courses is a natural consequence of a high level of excitement on the research frontier as newer techniques, such as numerical and symbolic computer systems, dynamical systems, and chaos, mix with and reinforce the traditional methods of applied mathematics. Thus, the purpose of this textbook series is to meet the current and future needs of these advances and encourage the teaching of new courses.

TAM will publish textbooks suitable for use in advanced undergraduate and beginning graduate courses, and will complement the *Applied Mathematical Sciences* (*AMS*) series, which will focus on advanced textbooks and research level monographs.

Preface

A proper understanding of the theory of boundary value problems, as opposed to a knowledge of techniques for solving specific problems or classes of problems, requires some background in functional analysis. The same is true of the finite element method: there is much that can be learned and practised – for example, the basic theory of the method, computational aspects, and so on – without knowledge of even the most basic notions of functional analysis. But for anyone wishing to gain a proper understanding of qualitative aspects of boundary value problems, or of aspects of the finite element method such as those that lead to the development of error estimates, some background in functional analysis is an essential prerequisite.

The issue of an adequate mathematical background is somewhat more straightforward in the case of students of mathematics who have taken courses in real and complex analysis, followed by a course in functional analysis. Such students are ideally equipped to follow courses that deal with existence theory for boundary value problems, and with qualitative aspects of the finite element method. This text has arisen out of a recognition, though, that there are many students, researchers, and practitioners who have not been exposed to the kind of mathematical background just referred to, but who nevertheless wish to become acquainted with the basic notions of functional analysis and its application to the kinds of problems that arise typically in physics and engineering.

Up to the mid-1970s the availability of source material to which such individuals could refer, at least in the English language, was limited almost entirely to the standard texts on real and functional analysis, written by mathematicians for mathematicians having the standard background. The task facing the engineer or applied scientist was thus quite daunting. Fortunately the situation has progressed markedly since then. There is now available a wide range of texts that present functional analysis, often with one or more applications taken from engineering and physics, in a manner accessible to readers not having the standard prerequisites. The styles differ, sometimes quite considerably, from one text to another, although this is not a bad thing given the diversity of interests and backgrounds of the potential readership.

This text is a further addition to the set of books that present functional analysis and its applications to nonspecialists. The approach taken is, first, to assume that readers have no more by way of relevant background than elementary courses in linear algebra, vector analysis, and differential equations, and wish to learn the elements of linear functional analysis.

The book begins with an introductory chapter, which is somewhat in the nature of a prologue, and which presents in mostly descriptive form a motivation for studying functional analysis from the viewpoint of those involved in the study of problems from physics and engineering. The remainder of the book is then divided into parts: Part I is devoted to linear functional analysis, Part II to an introduction to elliptic boundary value problems, and Part III comprises a study of the finite element method.

Two applications are treated in detail in this text: elliptic boundary value problems and the finite element method. In both cases any prior exposure to these areas will represent an advantage to those using this book; indeed, it is expected that such prior exposure will in many cases have provided the motivation to study the material presented here. The presentation of these applications starts more or less at the beginning, so that those having no background in these areas could use this text to acquire such background. On the other hand, it may be the case that the motivation to learn functional analysis arises from an interest in an area of application other than those treated in this text. Such readers might well prefer to focus on Part I of the book.

The incorporation of applications and other illustrative material is approached in two distinct ways. In Part I of the book new concepts, often of an abstract nature, are rendered more accessible by the copious use of concrete worked examples. There is little reference in this part of the book to applications in physics and engineering, for the simple reason that such examples are less well suited to laying bare the essential features of the many new concepts that accompany any introduction to functional analysis. In Parts II and III, it is appropriate and desirable to illustrate abstract concepts by recourse to concrete problems and examples taken from physics and engineering, and this is the approach taken here. I have used as examples problems such as heat conduction, as well as problems in solid and structural mechanics – elasticity, beams, and plates – and return regularly in Parts II and III to these examples in order to motivate and illustrate aspects of the theory of elliptic boundary value problems, and finite elements.

The style adopted in this text differs from that to be found in most texts on analysis, in that it is adapted to the goal of making the subject matter accessible. Thus proofs are sometimes omitted when these are felt to shed little additional light on the relevant topic. It will also be found in places that the presentation of detailed mathematical argument is eschewed in favor of a more descriptive approach, again for the purpose of rendering the material more accessible.

In addition to the many examples, each chapter ends with a collection of exercises for the reader. Some of these consolidate material presented in the chapters, and many exercises serve the purpose of amplification and supplementation. In both cases the exercises are to be regarded as an essential component of the text. Solutions to most of the exercises are presented at the end of the text.

Many individuals have assisted, in various ways, in the completion of this book. I am particularly grateful to Christiaan le Roux, Jean Lubuma, and Sizwe Mabizela, all of whom gave most generously of their time in reading and criticizing a preliminary version of the text. They offered detailed criticism on aspects of style and substance, and pointed out a number of errors. David Davidson organized a study group which worked through most of the book; I found the comments of this group of engineering scientists very helpful indeed. Weimin Han deserves special thanks for his constructive suggestions; so too does Brendt Wohlberg, who offered many suggestions for improving the text, located errors, and also provided me with valuable advice on the preparation of figures by computer. Shaun Courtney's expert guidance in the mysteries of Unix, and his very willing assistance with a variety of LaTeX problems, are much appreciated. Most of the figures were prepared by Bruce Bassett and Jill Goode, while Diane Laugksch assisted me in the typing of drafts of sections of the book. I am most grateful to these individuals for their cheerful assistance.

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Contents