Applied Mathematical Sciences

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With 10 Illustrations

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Contents

Preface

Our purpose in writing this book is to provide computer algebra programs which implement a number of popular perturbation methods. For each perturbation method, we present an introduction to the method, a couple of example problems, sample runs of the computer algebra programs and complete program listings.

In addition, we include examples of various elementary bifurcations, such as folds, pitchforks and Hopf bifurcations. These arise in the example problems. Specifically, we treat Hopf bifurcations in autonomous nonlinear systems via Lindstedt's method, the construction of center manifolds for simple, degenerate and nilpotent bifurcations in ordinary differential equations, the determination of normal forms for Hopf bifurcations and Takens-Bogdanov bifurcations, and averaging for autonomous and nonautonomous systems. Further, we use Lie transforms to determine normal forms in Hamiltonian systems. Bifurcation in partial differential equations, such as reaction diffusion equations or the Bernard convection problem, are treated via Liapunov-Schmidt reduction.

Moreover, we offer comparisons of the various methods. We compare averaging with normal forms, Liapunov-Schmidt reduction with center manifold reduction, Lindstedt's method with normal form calculations, and so on. To help in making the comparisons we frequently treat the same problem by two or more methods. E.g., we derive the Hopf bifurcation formula both by Lindstedt's method as well as via normal forms.

Our motivation for applying computer algebra to perturbation problems comes from the nature of the computations involved in these kinds of problems.

The massive algebra usually required to obtain detailed results is more quickly and more accurately accomplished by computer than by hand. Since our emphasis is on computation, we have dropped mathematical rigor in favor of intelligibility of the computational methods. However, we have provided the reader with references to standard mathematical textbooks or research papers.

The book assumes a knowledge of mathematics through a first year graduate course in applied mathematics. We have chosen the computer algebra system MACSYMA because it is popular and easy to learn, and some familiarity with MACSYMA is desirable [35]. For the reader who has no experience with MACSYMA we have provided a short introduction in the Appendix.

This book is perhaps best read in front of a computer terminal running MACSYMA. The reader could then enter the programs in this book as BATCH files, and run them on the sample problems. By examining the value of intermediate variables, greater understanding can be gained as to how the methods and programs work. Moreover, we hope that these programs will be useful utilities to research workers in applied mathematics. A note of caution has to be added: As the computational complexity of a problem is increased, e.g. by increasing the number of parameters or the number of equations or the order of truncation, there will come a point where the programs in this book will cease to work, either because of running out of memory or taking too long to run. We suggest that in such cases the reader may extend the usefulness of the program by tailoring it to fit the particular problem at hand.

We have tested the programs in this book on the following versions of MACSYMA: Eunice MACSYMA 308.2 on a VAX 8500 and MACSYMA 310.35 on a SYMBOLICS 3670. The timings which are given at the end of each run are machine dependent and approximate. Even on the same machine, the time for a given run will vary considerably due to "garbage collections" and other aspects of the LISP environment which are invisible to the user. While we have tried to design the programs to run efficiently, the inventive reader can probably improve upon our schemes and is encouraged to do so.

We offer to send an electronic file containing the programs to those readers who have access to BITNET. Our BITNET addresses are currently: RHRY@CRNLVAX5 (for RHR) and URBY@CRNLVAX5 (for DA).

We wish to thank the following people for the help they gave us: Fritz Busse, Tapesh Chakraborty, P.Y. Chen, Vincent Coppola, Richard Cushman, Gerhard Dangelmayr, Larry Fresinsky, Ruediger Gebauer, Jim Geer, Mohammed Golnaraghi, John Guckenheimer, Werner Guettinger, Peter Haug, Tim Healey, Phil Holmes, Herbert Hui, Bill Keith, Jon Len, Alexander Mielke, Frank Moon, Martin Neveling, Ken Ridley, Jan Sanders.

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