Svetlozar T. Rachev Ludger Rüschendorf

Mass Transportation Problems

Volume I: Theory

Springer

Svetlozar T. Rachev
Department of Statistics
University of California
Santa Barbara, CA 93106
USA

Ludger Rüschendorf
Institut für Mathematische Stochastik
University of Freiburg
79014 Freiburg
Germany

Series Editors

J. Gani
Stochastic Analysis
Group, CMA
Australian National
University
Canberra ACT 0200
Australia

C.C. Heyde
Stochastic Analysis
Group, CMA
Australian National
University
Canberra ACT 0200
Australia

T.G. Kurtz
Department of
Mathematics
University of Wisconsin
480 Lincoln Drive
Madison, WI 53706
USA

Library of Congress Cataloging-in-Publication Data
Rachev, S.T. (Svetlozar Todorov)
Mass transportation problems / Svetlozar T. Rachev, Ludger Rüschendorf.
p. cm. – (Probability and its applications)
Includes bibliographical references and indexes.
Contents: vol. 1. Theory – vol. 2. Applications.
ISBN 0-387-98350-3 (vol. 1 : hardcover). – ISBN 0-387-98352-X (vol. 2 : hardcover)
1. Transportation problems (Programming) I. Rüschendorf, Ludger, 1948– . II. Title. III. Series: Springer series in statistics. Probability and its applications.
QA402.6.R33 1998
519.7'2–dc21 97-34593

© 1998 by the Applied Probability Trust.
All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer-Verlag New York, Inc., 175 Fifth Avenue, New York, NY 10010, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.
The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

ISBN 0-387-98350-3 Springer-Verlag New York Berlin Heidelberg SPIN 10490744

Contents to Volume I

Preface to Volume I **vii**

Preface to Volume II xv

1 Introduction **1**

1.1 Mass Transportation Problems in Probability Theory . . . 1

1.2 Specially Structured Transportation Problems 21

1.3 Two Examples of the Interplay Between Continuous and Discrete MTPs 23

1.4 Stochastic Applications 27

2 The Monge–Kantorovich Problem **57**

2.1 The Multivariate Monge–Kantorovich Problem: An Introduction 58

2.2 Primal and Dual Monge–Kantorovich Functionals 64

2.3 Duality Theorems in a Topological Setting 76

2.4 General Duality Theorem 82

2.5 Duality Theorems with Metric Cost Functions 86

2.6 Dual Representation for L_p-Minimal Metrics 96

3 Explicit Results for the Monge–Kantorovich Problem 107

3.1 The One-Dimensional Case 107

3.2 The Convex Case 112

3.3 The General Case 123

3.4 An Extension of the Kantorovich L_2-Minimal Problem . . 132

3.5 Maximum Probability of Sets, Maximum of Sums, and Stochastic Order 144

3.6 Hoeffding–Fréchet Bounds 151

3.7 Bounds for the Total Transportation Cost 158

4 Duality Theory for Mass Transfer Problems 161

4.1 Duality in the Compact Case 161

4.2 Cost Functions with Triangle Inequality 172

4.3 Reduction Theorems 190

4.4 Proofs of the Main Duality Theorems and a Discussion . . 207

4.5 Duality Theorems for Noncompact Spaces 219

4.6 Infinite Linear Programs 241

4.6.1 Duality Theory for an Abstract Scheme of Infinite-Dimensional Linear Programs and Its Application to the Mass Transfer Problem 241

4.6.2 Duality Theorems for the Mass Transfer Problem with Given Marginals 245

4.6.3 Duality Theorem for a Marginal Problem with Additional Constraints of Moment-Type 251

4.6.4 Duality theorem for a Further Extremal Marginal Problem 258

4.6.5 Duality Theorem for a Nontopological Version of the Mass Transfer Problem 265

5 Applications of the Duality Theory 275

5.1 Mass Transfer Problem with a Smooth Cost Function—Explicit Solution 275

5.2 Extension and Approximate Extension Theorems 290

5.2.1 The Simplest Extension Theorem $\big($the Case $X = E(S)$ and $X_1 = E(S_1)\big)$ 290

5.2.2 Approximate Extension Theorems 292

5.2.3 Extension Theorems 295

5.2.4 A continuous selection theorem 302

5.3 Approximation Theorems 306
5.4 An Application of the Duality Theory to the Strassen Theorem 319
5.5 Closed Preorders and Continuous Utility Functions 322
5.5.1 Statement of the Problem and the Idea of the Duality Approach 322
5.5.2 Functionally Closed Preorders 324
5.5.3 Two Generalizations of the Debreu Theorem 329
5.5.4 The Case of a Locally Compact Space 335
5.5.5 Varying preorders and a universal utility theorem 337
5.5.6 Functionally Closed Preorders and Strong Stochastic Dominance 341
5.6 Further Applications to Utility Theory 344
5.6.1 Preferences That Admit Lipschitz or Continuous Utility Functions 344
5.6.2 Application to Choice Theory in Mathematical Economics . 352
5.7 Applications to Set-Valued Dynamical Systems 354
5.7.1 Compact-Valued Dynamical Systems: Quasiperiodic Points 354
5.7.2 Compact-Valued Dynamical Systems: Asymptotic Behavior of Trajectories 358
5.7.3 A Dynamic Optimization Problem 363
5.8 Compensatory Transfers and Action Profiles 367

6 Mass Transshipment Problems and Ideal Metrics 371
6.1 Kantorovich–Rubinstein Problems with Constraints 372
6.2 Constraints on the κth Difference of Marginals 383
6.3 The General Case 402
6.4 Minimality of Ideal Metrics 414

References 429

Abbreviations 473

Symbols 475

Index 487