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Method of Difference Potentials and Its Applications

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Dedicated to the memory of my parents
Solomon Abramovich Ryaben'kii and
Berta Pavlovna Ryaben'kaya

Preface to the English Edition

The method of difference potentials (MDP) was proposed in [1]–[8] and significantly developed in [9]–[101] and some other works.

The present book describes the current state of the art in the method of difference potentials and is a revised and essentially supplemented version of the author's first book devoted to this method, which was published by "Nauka" in 1987 [100].

This monograph deals with the MDP apparatus and several of its applications, particularly to the following problems:

1. the numerical solution of interior and exterior boundary-value problems for systems of partial differential equations;

2. the construction of conditions at the artificial boundary of the computational domain, which equivalently replace the equations and conditions at infinity in stationary problems of gas flow past immersed bodies as well as in some other steady-state problems;

3. the spectral approach to the construction of artificial boundary conditions replacing the equations of propagation of physical fields outside the computational domain containing perturbation sources;

4. the construction of artificial boundary conditions on the boundary of the computational domain for numerically solving the scattering problems in large time in a neighborhood of a fixed or a moving scatterer;

5. the statement and solution of stationary mathematical problems of the active shielding of a given subdomain from the influence of perturbation sources located outside the screened subdomain.

The new possibilities provided by the method of difference potentials originate from the fact that this method combines several advantages of the classical Cauchy-type integral from the theory of analytic functions and the universality of difference schemes.

The book can be useful to readers interested in different fields and pursuing different goals.

To become acquainted with the method of difference potentials, it suffices to read the contents and to study the introduction where the basic ideas, constructions, and advantages of the method of difference potentials are discussed and illustrated by model examples related to the Poisson equation. This will give a full glimpse of the method of difference potentials.

The reader who is interested in one of the above-listed problems or in any other application studied in the book can pass, after the introduction, directly to a corresponding chapter from Parts IV–VIII of the book. Each of these parts contains the appropriate theoretical material from Parts I–III, which is necessary for particular applications and is given without proofs in the most convenient form. Parts I–III of the book are intended for the reader who is interested in the general concepts of the method of difference potentials and their theoretical justification, as well as in some new applications of the method of difference potentials.

Moreover, Parts I–III can be of interest in themselves to specialists in the field of the qualitative theory of partial differential equations and boundary integral and pseudodifferential equations. The point is that in these sections of the book the potentials, projections, and boundary Calderón–Seeley equations are modified. The author carried out these modifications and generalizations in order to obtain the differential closures of MDP constructions as the grid step-size tends to zero. However, they are also meaningful beyond the framework of MDP applications, just as concepts from the theory of partial differential equations.

The studies described in the book were chiefly carried out at the M. V. Keldysh Institute of Applied Mathematics of the Russian Academy of Sciences, which bears the name of its organizer, at the Department of Computational Mathematics of the Moscow Institute of Physics and Technology, at the Mechanical Engineering Research Institute of the Russian Academy of Sciences, as well as at the Institute for Mathematical Modeling of the Russian Academy of Sciences, at Tel Aviv University, and at ICASE, NASA Langley Research Center.

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It is a pleasure for me to particularly thank Professor S. V. Tsynkov for his active collaboration, which has become especially fruitful and versatile soon after S. V. Tsynkov started his scientific work in his student years at the Moscow Institute of Physics and Technology about 15 years ago.

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Moscow, June 2001 *The author*

From the Preface to the Russian Edition

The method of difference potentials (MDP) allows one to find numerical solutions of many problems related to partial differential equations. The method of difference potentials was proposed by the author in his doctorate (D. Sc.) thesis in 1969. In the succeeding years, the method of difference potentials was significantly developed by the author and his pupils who were graduate and postgraduate students at the Moscow Institute of Physics and Technology working under the author's guidance: A. Ya. Belyankov, M. Yu. Lokhanov, A. A. Reznik, I. L. Sofronov, A. M. Fedorovskii, and D. I. Yanushevich.

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The author wishes to thank the Board of Directors of the Institute of Applied Mathematics of the USSR Academy of Sciences, which nowadays bears the name of M. V. Keldysh, where the author has carried out the bulk of the research work presented in this monograph.

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Moscow, 1987 *The author*

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