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Numerical Mathematics

With 134 Illustrations

Springer

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Series Preface

Mathematics is playing an ever more important role in the physical and biological sciences, provoking a blurring of boundaries between scientific disciplines and a resurgence of interest in the modern as well as the classical techniques of applied mathematics. This renewal of interest, both in research and teaching, has led to the establishment of the series: *Texts in Applied Mathematics (TAM)*.

The development of new courses is a natural consequence of a high level of excitement on the research frontier as newer techniques, such as numerical and symbolic computer systems, dynamical systems, and chaos, mix with and reinforce the traditional methods of applied mathematics. Thus, the purpose of this textbook series is to meet the current and future needs of these advances and encourage the teaching of new courses.

TAM will publish textbooks suitable for use in advanced undergraduate and beginning graduate courses, and will complement the *Applied Mathematical Sciences (AMS)* series, which will focus on advanced textbooks and research level monographs.

Preface

Numerical mathematics is the branch of mathematics that proposes, develops, analyzes and applies methods from scientific computing to several fields including analysis, linear algebra, geometry, approximation theory, functional equations, optimization and differential equations. Other disciplines such as physics, the natural and biological sciences, engineering, and economics and the financial sciences frequently give rise to problems that need scientific computing for their solutions.

As such, numerical mathematics is the crossroad of several disciplines of great relevance in modern applied sciences, and can become a crucial tool for their qualitative and quantitative analysis. This role is also emphasized by the continual development of computers and algorithms, which make it possible nowadays, using scientific computing, to tackle problems of such a large size that real-life phenomena can be simulated providing accurate responses at affordable computational cost.

The corresponding spread of numerical software represents an enrichment for the scientific community. However, the user has to make the correct choice of the method (or the algorithm) which best suits the problem at hand. As a matter of fact, no black-box methods or algorithms exist that can effectively and accurately solve all kinds of problems.

One of the purposes of this book is to provide the mathematical foundations of numerical methods, to analyze their basic theoretical properties (stability, accuracy, computational complexity), and demonstrate their performances on examples and counterexamples which outline their pros and cons. This is done using the MATLAB® [1] software environment. This choice satisfies the two fundamental needs of user-friendliness and widespread diffusion, making it available on virtually every computer.

Every chapter is supplied with examples, exercises and applications of the discussed theory to the solution of real-life problems. The reader is thus in the ideal condition for acquiring the theoretical knowledge that is required to make the right choice among the numerical methodologies and make use of the related computer programs.

This book is primarily addressed to undergraduate students, with particular focus on the degree courses in Engineering, Mathematics, Physics and Computer Science. The attention which is paid to the applications and the related development of software makes it valuable also for graduate students, researchers and users of scientific computing in the most widespread professional fields.

The content of the volume is organized into four parts and 13 chapters.

Part I comprises two chapters in which we review basic linear algebra and introduce the general concepts of consistency, stability and convergence of a numerical method as well as the basic elements of computer arithmetic.

Part II is on numerical linear algebra, and is devoted to the solution of linear systems (Chapters 3 and 4) and eigenvalues and eigenvectors computation (Chapter 5).

We continue with Part III where we face several issues about functions and their approximation. Specifically, we are interested in the solution of nonlinear equations (Chapter 6), solution of nonlinear systems and optimization problems (Chapter 7), polynomial approximation (Chapter 8) and numerical integration (Chapter 9).

Part IV, which is the more demanding as a mathematical background, is concerned with approximation, integration and transforms based on orthogonal polynomials (Chapter 10), solution of initial value problems (Chapter 11), boundary value problems (Chapter 12) and initial-boundary value problems for parabolic and hyperbolic equations (Chapter 13).

Part I provides the indispensable background. Each of the remaining Parts has a size and a content that make it well suited for a semester course.

A guideline index to the use of the numerous MATLAB Programs developed in the book is reported at the end of the volume. These programs are also available at the web site address:

http://www1.mate.polimi.it/~calnum/programs.html

For the reader's ease, any code is accompanied by a brief description of its input/output parameters.

We express our thanks to the staff at Springer-Verlag New York for their expert guidance and assistance with editorial aspects, as well as to Dr.

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[1] MATLAB is a registered trademark of The MathWorks, Inc.

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