

Texts in Applied Mathematics **37**

Editors

J.E. Marsden
L. Sirovich
M. Golubitsky
W. Jäger

Advisors

G. Iooss
P. Holmes
D. Barkley
M. Dellnitz
P. Newton

Texts in Applied Mathematics

1. *Sirovich*: Introduction to Applied Mathematics.
2. *Wiggins*: Introduction to Applied Nonlinear Dynamical Systems and Chaos.
3. *Hale/Koçak*: Dynamics and Bifurcations.
4. *Chorin/Marsden*: A Mathematical Introduction to Fluid Mechanics, 3rd ed.
5. *Hubbard/West*: Differential Equations: A Dynamical Systems Approach: Ordinary Differential Equations.
6. *Sontag*: Mathematical Control Theory: Deterministic Finite Dimensional Systems, 2nd ed.
7. *Perko*: Differential Equations and Dynamical Systems, 2nd ed.
8. *Seaborn*: Hypergeometric Functions and Their Applications.
9. *Pipkin*: A Course on Integral Equations.
10. *Hoppensteadt/Peskin*: Mathematics in Medicine and the Life Sciences.
11. *Braun*: Differential Equations and Their Applications, 4th ed.
12. *Stoer/Bulirsch*: Introduction to Numerical Analysis, 2nd ed.
13. *Renardy/Rogers*: A First Graduate Course in Partial Differential Equations.
14. *Banks*: Growth and Diffusion Phenomena: Mathematical Frameworks and Applications.
15. *Brenner/Scott*: The Mathematical Theory of Finite Element Methods.
16. *Van de Velde*: Concurrent Scientific Computing.
17. *Marsden/Ratiu*: Introduction to Mechanics and Symmetry, 2nd ed.
18. *Hubbard/West*: Differential Equations: A Dynamical Systems Approach: Higher-Dimensional Systems.
19. *Kaplan/Glass*: Understanding Nonlinear Dynamics.
20. *Holmes*: Introduction to Perturbation Methods.
21. *Curtain/Zwart*: An Introduction to Infinite-Dimensional Linear Systems Theory.
22. *Thomas*: Numerical Partial Differential Equations: Finite Difference Methods.
23. *Taylor*: Partial Differential Equations: Basic Theory.
24. *Merkin*: Introduction to the Theory of Stability of Motion.
25. *Naber*: Topology, Geometry, and Gauge Fields: Foundations.
26. *Polderman/Willems*: Introduction to Mathematical Systems Theory: A Behavioral Approach.
27. *Reddy*: Introductory Functional Analysis with Applications to Boundary-Value Problems and Finite Elements.
28. *Gustafson/Wilcox*: Analytical and Computational Methods of Advanced Engineering Mathematics.
29. *Tveito/Winther*: Introduction to Partial Differential Equations: A Computational Approach.
30. *Gasquet/Witomski*: Fourier Analysis and Applications: Filtering, Numerical Computation, Wavelets.
31. *Brémaud*: Markov Chains: Gibbs Fields, Monte Carlo Simulation, and Queues.
32. *Durrant*: Numerical Methods for Wave Equations in Geophysical Fluid Dynamics.

(continued after index)

Alfio Quarteroni
Fausto Saleri

Riccardo Sacco

Numerical Mathematics

With 134 Illustrations



Springer

Alfio Quarteroni
Department of Mathematics
Ecole Polytechnique
Fédérale de Lausanne
CH-1015 Lausanne
Switzerland
alfio.quarteroni@epfl.ch

Riccardo Sacco
Dipartimento di Matematica
Politecnico di Milano
Piazza Leonardo da Vinci 32
20133 Milan
Italy
ricsac@mate.polimi.it

Fausto Saleri
Dipartimento di Matematica,
"F. Enriques"
Università degli Studi di
Milano
Via Saldini 50
20133 Milan
Italy
fausto.saleri@unimi.it

Series Editors

J.E. Marsden
Control and Dynamical Systems, 107–81
California Institute of Technology
Pasadena, CA 91125
USA

L. Sirovich
Division of Applied Mathematics
Brown University
Providence, RI 02912
USA

M. Golubitsky
Department of Mathematics
University of Houston
Houston, TX 77204-3476
USA

W. Jäger
Department of Applied Mathematics
Universität Heidelberg
Im Neuenheimer Feld 294
69120 Heidelberg
Germany

Mathematics Subject Classification (1991): 15-01, 34-01, 35-01, 65-01

Library of Congress Cataloging-in-Publication Data
Quarteroni, Alfio.

Numerical mathematics/Alfio Quarteroni, Riccardo Sacco, Fausto Saleri.

p. cm. — (Texts in applied mathematics; 37)

Includes bibliographical references and index.

ISBN 978-1-4757-7394-1 ISBN 978-0-387-22750-4 (eBook)

DOI 10.1007/978-0-387-22750-4

1. Numerical analysis. I. Sacco, Riccardo. II. Saleri, Fausto. III. Title. IV. Series.

I. Title. II. Series.

QA297.Q83 2000

519.4—dc21

99-059414

Printed on acid-free paper.

© 2000 Springer Science+Business Media New York
Originally published by Springer-Verlag New York, Inc. in 2000
Softcover reprint of the hardcover 1st edition 2000

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher Springer Science+Business Media, LLC, except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

Production managed by Frank McGuckin; manufacturing supervised by Jeffrey Taub.
Camera-ready copy prepared from the authors' LaTeX files using Springer's svsing.sty macro.

9 8 7 6 5 4 3 2 1

ISBN 978-1-4757-7394-1

SPIN 10747955

Series Preface

Mathematics is playing an ever more important role in the physical and biological sciences, provoking a blurring of boundaries between scientific disciplines and a resurgence of interest in the modern as well as the classical techniques of applied mathematics. This renewal of interest, both in research and teaching, has led to the establishment of the series: *Texts in Applied Mathematics (TAM)*.

The development of new courses is a natural consequence of a high level of excitement on the research frontier as newer techniques, such as numerical and symbolic computer systems, dynamical systems, and chaos, mix with and reinforce the traditional methods of applied mathematics. Thus, the purpose of this textbook series is to meet the current and future needs of these advances and encourage the teaching of new courses.

TAM will publish textbooks suitable for use in advanced undergraduate and beginning graduate courses, and will complement the *Applied Mathematical Sciences (AMS)* series, which will focus on advanced textbooks and research level monographs.

Preface

Numerical mathematics is the branch of mathematics that proposes, develops, analyzes and applies methods from scientific computing to several fields including analysis, linear algebra, geometry, approximation theory, functional equations, optimization and differential equations. Other disciplines such as physics, the natural and biological sciences, engineering, and economics and the financial sciences frequently give rise to problems that need scientific computing for their solutions.

As such, numerical mathematics is the crossroad of several disciplines of great relevance in modern applied sciences, and can become a crucial tool for their qualitative and quantitative analysis. This role is also emphasized by the continual development of computers and algorithms, which make it possible nowadays, using scientific computing, to tackle problems of such a large size that real-life phenomena can be simulated providing accurate responses at affordable computational cost.

The corresponding spread of numerical software represents an enrichment for the scientific community. However, the user has to make the correct choice of the method (or the algorithm) which best suits the problem at hand. As a matter of fact, no black-box methods or algorithms exist that can effectively and accurately solve all kinds of problems.

One of the purposes of this book is to provide the mathematical foundations of numerical methods, to analyze their basic theoretical properties (stability, accuracy, computational complexity), and demonstrate their performances on examples and counterexamples which outline their pros

and cons. This is done using the MATLAB[®]¹ software environment. This choice satisfies the two fundamental needs of user-friendliness and widespread diffusion, making it available on virtually every computer.

Every chapter is supplied with examples, exercises and applications of the discussed theory to the solution of real-life problems. The reader is thus in the ideal condition for acquiring the theoretical knowledge that is required to make the right choice among the numerical methodologies and make use of the related computer programs.

This book is primarily addressed to undergraduate students, with particular focus on the degree courses in Engineering, Mathematics, Physics and Computer Science. The attention which is paid to the applications and the related development of software makes it valuable also for graduate students, researchers and users of scientific computing in the most widespread professional fields.

The content of the volume is organized into four parts and 13 chapters.

Part I comprises two chapters in which we review basic linear algebra and introduce the general concepts of consistency, stability and convergence of a numerical method as well as the basic elements of computer arithmetic.

Part II is on numerical linear algebra, and is devoted to the solution of linear systems (Chapters 3 and 4) and eigenvalues and eigenvectors computation (Chapter 5).

We continue with Part III where we face several issues about functions and their approximation. Specifically, we are interested in the solution of nonlinear equations (Chapter 6), solution of nonlinear systems and optimization problems (Chapter 7), polynomial approximation (Chapter 8) and numerical integration (Chapter 9).

Part IV, which is the more demanding as a mathematical background, is concerned with approximation, integration and transforms based on orthogonal polynomials (Chapter 10), solution of initial value problems (Chapter 11), boundary value problems (Chapter 12) and initial-boundary value problems for parabolic and hyperbolic equations (Chapter 13).

Part I provides the indispensable background. Each of the remaining Parts has a size and a content that make it well suited for a semester course.

A guideline index to the use of the numerous MATLAB Programs developed in the book is reported at the end of the volume. These programs are also available at the web site address:

<http://www1.mate.polimi.it/~calnum/programs.html>

For the reader's ease, any code is accompanied by a brief description of its input/output parameters.

We express our thanks to the staff at Springer-Verlag New York for their expert guidance and assistance with editorial aspects, as well as to Dr.

¹MATLAB is a registered trademark of The MathWorks, Inc.

Martin Peters from Springer-Verlag Heidelberg and Dr. Francesca Bonadei from Springer-Italia for their advice and friendly collaboration all along this project.

We gratefully thank Professors L. Gastaldi and A. Valli for their useful comments on Chapters 12 and 13.

We also wish to express our gratitude to our families for their forbearance and understanding, and dedicate this book to them.

Lausanne, Switzerland
Milan, Italy
Milan, Italy
January 2000

Alfio Quarteroni
Riccardo Sacco
Fausto Saleri

Contents

Series Preface	v
Preface	vii
PART I: Getting Started	
1. Foundations of Matrix Analysis	1
1.1 Vector Spaces	1
1.2 Matrices	3
1.3 Operations with Matrices	5
1.3.1 Inverse of a Matrix	6
1.3.2 Matrices and Linear Mappings	7
1.3.3 Operations with Block-Partitioned Matrices	7
1.4 Trace and Determinant of a Matrix	8
1.5 Rank and Kernel of a Matrix	9
1.6 Special Matrices	10
1.6.1 Block Diagonal Matrices	10
1.6.2 Trapezoidal and Triangular Matrices	11
1.6.3 Banded Matrices	11
1.7 Eigenvalues and Eigenvectors	12
1.8 Similarity Transformations	14
1.9 The Singular Value Decomposition (SVD)	16
1.10 Scalar Product and Norms in Vector Spaces	17
1.11 Matrix Norms	21

1.11.1	Relation Between Norms and the Spectral Radius of a Matrix	25
1.11.2	Sequences and Series of Matrices	26
1.12	Positive Definite, Diagonally Dominant and M-Matrices	27
1.13	Exercises	30
2.	Principles of Numerical Mathematics	33
2.1	Well-Posedness and Condition Number of a Problem	33
2.2	Stability of Numerical Methods	37
2.2.1	Relations Between Stability and Convergence	40
2.3	<i>A priori</i> and <i>a posteriori</i> Analysis	41
2.4	Sources of Error in Computational Models	43
2.5	Machine Representation of Numbers	45
2.5.1	The Positional System	45
2.5.2	The Floating-Point Number System	46
2.5.3	Distribution of Floating-Point Numbers	49
2.5.4	IEC/IEEE Arithmetic	49
2.5.5	Rounding of a Real Number in Its Machine Representation	50
2.5.6	Machine Floating-Point Operations	52
2.6	Exercises	54
 PART II: Numerical Linear Algebra		
3.	Direct Methods for the Solution of Linear Systems	57
3.1	Stability Analysis of Linear Systems	58
3.1.1	The Condition Number of a Matrix	58
3.1.2	Forward <i>a priori</i> Analysis	60
3.1.3	Backward <i>a priori</i> Analysis	63
3.1.4	<i>A posteriori</i> Analysis	64
3.2	Solution of Triangular Systems	65
3.2.1	Implementation of Substitution Methods	65
3.2.2	Rounding Error Analysis	67
3.2.3	Inverse of a Triangular Matrix	67
3.3	The Gaussian Elimination Method (GEM) and LU Factorization	68
3.3.1	GEM as a Factorization Method	72
3.3.2	The Effect of Rounding Errors	76
3.3.3	Implementation of LU Factorization	77
3.3.4	Compact Forms of Factorization	78
3.4	Other Types of Factorization	79
3.4.1	LDM ^T Factorization	79
3.4.2	Symmetric and Positive Definite Matrices: The Cholesky Factorization	80
3.4.3	Rectangular Matrices: The QR Factorization	82

3.5	Pivoting	85
3.6	Computing the Inverse of a Matrix	89
3.7	Banded Systems	90
	3.7.1 Tridiagonal Matrices	91
	3.7.2 Implementation Issues	92
3.8	Block Systems	93
	3.8.1 Block LU Factorization	94
	3.8.2 Inverse of a Block-Partitioned Matrix	95
	3.8.3 Block Tridiagonal Systems	95
3.9	Sparse Matrices	97
	3.9.1 The Cuthill-McKee Algorithm	98
	3.9.2 Decomposition into Substructures	100
	3.9.3 Nested Dissection	103
3.10	Accuracy of the Solution Achieved Using GEM	103
3.11	An Approximate Computation of $K(A)$	106
3.12	Improving the Accuracy of GEM	109
	3.12.1 Scaling	110
	3.12.2 Iterative Refinement	111
3.13	Undetermined Systems	112
3.14	Applications	115
	3.14.1 Nodal Analysis of a Structured Frame	115
	3.14.2 Regularization of a Triangular Grid	118
3.15	Exercises	121
4.	Iterative Methods for Solving Linear Systems	123
4.1	On the Convergence of Iterative Methods	123
4.2	Linear Iterative Methods	126
	4.2.1 Jacobi, Gauss-Seidel and Relaxation Methods	127
	4.2.2 Convergence Results for Jacobi and Gauss-Seidel Methods	129
	4.2.3 Convergence Results for the Relaxation Method	131
	4.2.4 <i>A priori</i> Forward Analysis	132
	4.2.5 Block Matrices	133
	4.2.6 Symmetric Form of the Gauss-Seidel and SOR Methods	133
	4.2.7 Implementation Issues	135
4.3	Stationary and Nonstationary Iterative Methods	136
	4.3.1 Convergence Analysis of the Richardson Method	137
	4.3.2 Preconditioning Matrices	139
	4.3.3 The Gradient Method	146
	4.3.4 The Conjugate Gradient Method	150
	4.3.5 The Preconditioned Conjugate Gradient Method	156
	4.3.6 The Alternating-Direction Method	158
4.4	Methods Based on Krylov Subspace Iterations	159
	4.4.1 The Arnoldi Method for Linear Systems	162

4.4.2	The GMRES Method	165
4.4.3	The Lanczos Method for Symmetric Systems	167
4.5	The Lanczos Method for Unsymmetric Systems	168
4.6	Stopping Criteria	171
4.6.1	A Stopping Test Based on the Increment	172
4.6.2	A Stopping Test Based on the Residual	174
4.7	Applications	174
4.7.1	Analysis of an Electric Network	174
4.7.2	Finite Difference Analysis of Beam Bending	177
4.8	Exercises	179
5.	Approximation of Eigenvalues and Eigenvectors	183
5.1	Geometrical Location of the Eigenvalues	183
5.2	Stability and Conditioning Analysis	186
5.2.1	<i>A priori</i> Estimates	186
5.2.2	<i>A posteriori</i> Estimates	190
5.3	The Power Method	192
5.3.1	Approximation of the Eigenvalue of Largest Module	192
5.3.2	Inverse Iteration	195
5.3.3	Implementation Issues	196
5.4	The QR Iteration	200
5.5	The Basic QR Iteration	201
5.6	The QR Method for Matrices in Hessenberg Form	203
5.6.1	Householder and Givens Transformation Matrices	204
5.6.2	Reducing a Matrix in Hessenberg Form	207
5.6.3	QR Factorization of a Matrix in Hessenberg Form	209
5.6.4	The Basic QR Iteration Starting from Upper Hessenberg Form	210
5.6.5	Implementation of Transformation Matrices	212
5.7	The QR Iteration with Shifting Techniques	215
5.7.1	The QR Method with Single Shift	215
5.7.2	The QR Method with Double Shift	218
5.8	Computing the Eigenvectors and the SVD of a Matrix	221
5.8.1	The Hessenberg Inverse Iteration	221
5.8.2	Computing the Eigenvectors from the Schur Form of a Matrix	221
5.8.3	Approximate Computation of the SVD of a Matrix	222
5.9	The Generalized Eigenvalue Problem	224
5.9.1	Computing the Generalized Real Schur Form	225
5.9.2	Generalized Real Schur Form of Symmetric-Definite Pencils	226
5.10	Methods for Eigenvalues of Symmetric Matrices	227
5.10.1	The Jacobi Method	227
5.10.2	The Method of Sturm Sequences	230

5.11 The Lanczos Method 233
 5.12 Applications 235
 5.12.1 Analysis of the Buckling of a Beam 236
 5.12.2 Free Dynamic Vibration of a Bridge 238
 5.13 Exercises 240

PART III: Around Functions and Functionals

6. Rootfinding for Nonlinear Equations 245
 6.1 Conditioning of a Nonlinear Equation 246
 6.2 A Geometric Approach to Rootfinding 248
 6.2.1 The Bisection Method 248
 6.2.2 The Methods of Chord, Secant and Regula Falsi
 and Newton's Method 251
 6.2.3 The Dekker-Brent Method 256
 6.3 Fixed-Point Iterations for Nonlinear Equations 257
 6.3.1 Convergence Results for
 Some Fixed-Point Methods 260
 6.4 Zeros of Algebraic Equations 261
 6.4.1 The Horner Method and Deflation 262
 6.4.2 The Newton-Horner Method 263
 6.4.3 The Muller Method 267
 6.5 Stopping Criteria 269
 6.6 Post-Processing Techniques for Iterative Methods 272
 6.6.1 Aitken's Acceleration 272
 6.6.2 Techniques for Multiple Roots 275
 6.7 Applications 276
 6.7.1 Analysis of the State Equation for a Real Gas 276
 6.7.2 Analysis of a Nonlinear Electrical Circuit 277
 6.8 Exercises 279

7. Nonlinear Systems and Numerical Optimization 281
 7.1 Solution of Systems of Nonlinear Equations 282
 7.1.1 Newton's Method and Its Variants 283
 7.1.2 Modified Newton's Methods 284
 7.1.3 Quasi-Newton Methods 288
 7.1.4 Secant-Like Methods 288
 7.1.5 Fixed-Point Methods 290
 7.2 Unconstrained Optimization 294
 7.2.1 Direct Search Methods 295
 7.2.2 Descent Methods 300
 7.2.3 Line Search Techniques 302
 7.2.4 Descent Methods for Quadratic Functions 304
 7.2.5 Newton-Like Methods for Function Minimization 307
 7.2.6 Quasi-Newton Methods 308

7.2.7	Secant-Like Methods	309
7.3	Constrained Optimization	311
7.3.1	Kuhn-Tucker Necessary Conditions for Nonlinear Programming	313
7.3.2	The Penalty Method	315
7.3.3	The Method of Lagrange Multipliers	317
7.4	Applications	319
7.4.1	Solution of a Nonlinear System Arising from Semiconductor Device Simulation	320
7.4.2	Nonlinear Regularization of a Discretization Grid	323
7.5	Exercises	325
8.	Polynomial Interpolation	327
8.1	Polynomial Interpolation	328
8.1.1	The Interpolation Error	329
8.1.2	Drawbacks of Polynomial Interpolation on Equally Spaced Nodes and Runge's Counterexample	330
8.1.3	Stability of Polynomial Interpolation	332
8.2	Newton Form of the Interpolating Polynomial	333
8.2.1	Some Properties of Newton Divided Differences	335
8.2.2	The Interpolation Error Using Divided Differences	337
8.3	Piecewise Lagrange Interpolation	338
8.4	Hermite-Birkoff Interpolation	341
8.5	Extension to the Two-Dimensional Case	343
8.5.1	Polynomial Interpolation	343
8.5.2	Piecewise Polynomial Interpolation	344
8.6	Approximation by Splines	348
8.6.1	Interpolatory Cubic Splines	349
8.6.2	B-Splines	353
8.7	Splines in Parametric Form	357
8.7.1	Bézier Curves and Parametric B-Splines	359
8.8	Applications	362
8.8.1	Finite Element Analysis of a Clamped Beam	363
8.8.2	Geometric Reconstruction Based on Computer Tomographies	366
8.9	Exercises	368
9.	Numerical Integration	371
9.1	Quadrature Formulae	371
9.2	Interpolatory Quadratures	373
9.2.1	The Midpoint or Rectangle Formula	373
9.2.2	The Trapezoidal Formula	375
9.2.3	The Cavalieri-Simpson Formula	377
9.3	Newton-Cotes Formulae	378
9.4	Composite Newton-Cotes Formulae	383

- 9.5 Hermite Quadrature Formulae 386
- 9.6 Richardson Extrapolation 387
 - 9.6.1 Romberg Integration 389
- 9.7 Automatic Integration 391
 - 9.7.1 Non Adaptive Integration Algorithms 392
 - 9.7.2 Adaptive Integration Algorithms 394
- 9.8 Singular Integrals 398
 - 9.8.1 Integrals of Functions with Finite
Jump Discontinuities 398
 - 9.8.2 Integrals of Infinite Functions 398
 - 9.8.3 Integrals over Unbounded Intervals 401
- 9.9 Multidimensional Numerical Integration 402
 - 9.9.1 The Method of Reduction Formula 403
 - 9.9.2 Two-Dimensional Composite Quadratures 404
 - 9.9.3 Monte Carlo Methods for
Numerical Integration 407
- 9.10 Applications 408
 - 9.10.1 Computation of an Ellipsoid Surface 408
 - 9.10.2 Computation of the Wind Action on a
Sailboat Mast 410
- 9.11 Exercises 412

**PART IV: Transforms, Differentiation
and Problem Discretization**

- 10. Orthogonal Polynomials in Approximation Theory 415**
 - 10.1 Approximation of Functions by Generalized Fourier Series 415
 - 10.1.1 The Chebyshev Polynomials 417
 - 10.1.2 The Legendre Polynomials 419
 - 10.2 Gaussian Integration and Interpolation 419
 - 10.3 Chebyshev Integration and Interpolation 424
 - 10.4 Legendre Integration and Interpolation 426
 - 10.5 Gaussian Integration over Unbounded Intervals 428
 - 10.6 Programs for the Implementation of Gaussian Quadratures 429
 - 10.7 Approximation of a Function in the Least-Squares Sense . 431
 - 10.7.1 Discrete Least-Squares Approximation 431
 - 10.8 The Polynomial of Best Approximation 433
 - 10.9 Fourier Trigonometric Polynomials 435
 - 10.9.1 The Gibbs Phenomenon 439
 - 10.9.2 The Fast Fourier Transform 440
 - 10.10 Approximation of Function Derivatives 442
 - 10.10.1 Classical Finite Difference Methods 442
 - 10.10.2 Compact Finite Differences 444
 - 10.10.3 Pseudo-Spectral Derivative 448
 - 10.11 Transforms and Their Applications 450

10.11.1	The Fourier Transform	450
10.11.2	(Physical) Linear Systems and Fourier Transform	453
10.11.3	The Laplace Transform	455
10.11.4	The Z-Transform	457
10.12	The Wavelet Transform	458
10.12.1	The Continuous Wavelet Transform	458
10.12.2	Discrete and Orthonormal Wavelets	461
10.13	Applications	463
10.13.1	Numerical Computation of Blackbody Radiation	463
10.13.2	Numerical Solution of Schrödinger Equation	464
10.14	Exercises	467
11.	Numerical Solution of Ordinary Differential Equations	469
11.1	The Cauchy Problem	469
11.2	One-Step Numerical Methods	472
11.3	Analysis of One-Step Methods	473
11.3.1	The Zero-Stability	475
11.3.2	Convergence Analysis	477
11.3.3	The Absolute Stability	479
11.4	Difference Equations	482
11.5	Multistep Methods	487
11.5.1	Adams Methods	490
11.5.2	BDF Methods	492
11.6	Analysis of Multistep Methods	492
11.6.1	Consistency	493
11.6.2	The Root Conditions	494
11.6.3	Stability and Convergence Analysis for Multistep Methods	495
11.6.4	Absolute Stability of Multistep Methods	499
11.7	Predictor-Corrector Methods	502
11.8	Runge-Kutta Methods	508
11.8.1	Derivation of an Explicit RK Method	511
11.8.2	Stepsize Adaptivity for RK Methods	512
11.8.3	Implicit RK Methods	514
11.8.4	Regions of Absolute Stability for RK Methods	516
11.9	Systems of ODEs	517
11.10	Stiff Problems	519
11.11	Applications	521
11.11.1	Analysis of the Motion of a Frictionless Pendulum	522
11.11.2	Compliance of Arterial Walls	523
11.12	Exercises	527
12.	Two-Point Boundary Value Problems	531
12.1	A Model Problem	531
12.2	Finite Difference Approximation	533

12.2.1	Stability Analysis by the Energy Method	534
12.2.2	Convergence Analysis	538
12.2.3	Finite Differences for Two-Point Boundary Value Problems with Variable Coefficients	540
12.3	The Spectral Collocation Method	542
12.4	The Galerkin Method	544
12.4.1	Integral Formulation of Boundary-Value Problems	544
12.4.2	A Quick Introduction to Distributions	546
12.4.3	Formulation and Properties of the Galerkin Method	547
12.4.4	Analysis of the Galerkin Method	548
12.4.5	The Finite Element Method	550
12.4.6	Implementation Issues	556
12.4.7	Spectral Methods	559
12.5	Advection-Diffusion Equations	560
12.5.1	Galerkin Finite Element Approximation	561
12.5.2	The Relationship Between Finite Elements and Finite Differences; the Numerical Viscosity	563
12.5.3	Stabilized Finite Element Methods	567
12.6	A Quick Glance to the Two-Dimensional Case	572
12.7	Applications	575
12.7.1	Lubrication of a Slider	575
12.7.2	Vertical Distribution of Spore Concentration over Wide Regions	576
12.8	Exercises	578

13. Parabolic and Hyperbolic Initial Boundary Value Problems	581
13.1 The Heat Equation	581
13.2 Finite Difference Approximation of the Heat Equation . .	584
13.3 Finite Element Approximation of the Heat Equation . . .	586
13.3.1 Stability Analysis of the θ -Method	588
13.4 Space-Time Finite Element Methods for the Heat Equation	593
13.5 Hyperbolic Equations: A Scalar Transport Problem	597
13.6 Systems of Linear Hyperbolic Equations	599
13.6.1 The Wave Equation	601
13.7 The Finite Difference Method for Hyperbolic Equations . .	602
13.7.1 Discretization of the Scalar Equation	602
13.8 Analysis of Finite Difference Methods	605
13.8.1 Consistency	605
13.8.2 Stability	605
13.8.3 The CFL Condition	606
13.8.4 Von Neumann Stability Analysis	608
13.9 Dissipation and Dispersion	611

13.9.1	Equivalent Equations	614
13.10	Finite Element Approximation of Hyperbolic Equations . .	618
13.10.1	Space Discretization with Continuous and Discontinuous Finite Elements	618
13.10.2	Time Discretization	620
13.11	Applications	623
13.11.1	Heat Conduction in a Bar	623
13.11.2	A Hyperbolic Model for Blood Flow Interaction with Arterial Walls	623
13.12	Exercises	625
	References	627
	Index of MATLAB Programs	643
	Index	647