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Quantum Machine Learning

Thinking and Exploration in Neural Network
Models for Quantum Science and Quantum
Computing



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Contents

1	Quantum Mechanics and Data-Driven Physics	1
1.1	Introduction	1
1.2	Quantum Feature Mapping and the Computational Model	2
1.3	Example 1: The First Supervised Learning by Superconducting Qubits	4
1.4	Example 2: Photonic Quantum Processors, Boson Sampling, and Quantum Advantage	4
1.5	Quantum State Representations	6
1.6	Pros and Cons of Quantum Neural Networks	7
1.7	Quantum Mechanics and Kernel Methods	9
1.8	More on Kernel Methods	12
1.9	Coding Example of Kernel Classification	14
1.10	Support Vector Machine: The Widest Street Approach*	16
1.11	The Link of Kernel Machines with the Perceptron	21
1.12	Kernel Classification of Nonlinearly Separable Data	21
1.13	Feature Mapping	24
1.14	The Drawbacks in Kernel Methods	26
1.15	Further Reading	26
	References	27
2	Kernelizing Quantum Mechanics	29
2.1	Introduction	29
2.2	The Computational Model by Quantum Feature Maps and Quantum Kernels	29
2.3	Quantum Feature Map by Coherent States	31
2.4	Quantum Classifier by Coherent States	32
2.5	How to Measure Scalar Products with Coherent States	35
2.6	Quantum Feature Map by Squeezed Vacuum	37
2.7	A Quantum Classifier by Squeezed Vacuum	38
2.8	Measuring Scalar Products with General States: The SWAP Test	40

2.9	Boson Sampling and Quantum Kernels	44
2.10	Universality of Quantum Feature Maps and the Reproducing Kernel Theorem*	46
2.11	Further Reading	49
	References	49
3	Qubit Maps	51
3.1	Introduction	51
3.2	Feature Maps with Qubits	51
3.3	Using Tensors with Single Qubits	52
3.3.1	One-Qubit Gate as a Tensor	53
3.4	More on Contravariant and Covariant Tensors in TensorFlow*	54
3.5	Hadamard Gate as a Tensor	55
3.6	Recap on Vectors and Matrices in TensorFlow*	56
3.6.1	Matrix-Vector Multiplication as a Contraction	58
3.6.2	Lists, Tensors, Row, and Column Vectors	59
3.7	One-Qubit Gates	61
3.8	Scalar Product	62
3.9	Two-Qubit Tensors	63
3.9.1	Further Remarks on the Tensor Indices	64
3.10	Two-Qubit Gates	66
3.10.1	Coding Two-Qubit Gates	69
3.10.2	CNOT Gate	70
3.10.3	CZ Gate	72
3.11	Quantum Feature Maps with a Single Qubit	74
3.11.1	Rotation Gates	75
3.12	Quantum Feature Maps with Two Qubits	76
3.13	Coding the Entangled Feature Map	78
3.13.1	The Single Pass Feature Map	79
3.13.2	The Iterated Feature Map	79
3.14	The Entangled Kernel and Its Measurement with Two Qubits ...	80
3.15	Classical Simulation of the Entangled Kernel	80
3.15.1	A Remark on the Computational Time	82
3.16	Further Reading	82
	References	83
4	One-Qubit Transverse-Field Ising Model and Variational Quantum Algorithms	85
4.1	Introduction	85
4.2	Mapping Combinatorial Optimization to the Ising Model	86
4.2.1	Number Partitioning	86
4.2.2	Maximum Cut	88
4.3	Quadratic Unconstrained Binary Optimization (QUBO)	89
4.4	Why Use a Quantum Computer for Combinatorial Optimization?	90

4.5	The Transverse-Field Ising Model	91
4.6	One-Qubit Transverse-Field Ising Model	92
4.6.1	$h = 0$	92
4.6.2	$J = 0$	93
4.7	Coding the One-Qubit Transverse-Field Ising Hamiltonian	93
4.7.1	The Hamiltonian as a <code>tensor</code>	93
4.7.2	Variational Ansatz as a <code>tensor</code>	95
4.8	A Neural Network Layer for the Hamiltonian	97
4.9	Training the One-Qubit Model	101
4.10	Further Reading	103
	References	104
5	Two-Qubit Transverse-Field Ising Model and Entanglement	105
5.1	Introduction	105
5.2	Two-Qubit Transverse-Field Ising Model	105
5.2.1	$h_0 = h_1 = 0$	106
5.2.2	Entanglement in the Ground State with No External Field	107
5.2.3	$h_0 = h_1 = h \neq 0$	107
5.3	Entanglement and Mixtures	108
5.4	The Reduced Density Matrix	110
5.5	The Partial Trace	112
5.6	One-Qubit Density Matrix Using Tensors	115
5.7	Coding the One-Qubit Density Matrix	117
5.8	Two-Qubit Density Matrix by Tensors	120
5.9	Coding the Two-Qubit Density Matrix	122
5.10	Partial Trace with <code>tensors</code>	124
5.11	Entropy of Entanglement	125
5.12	Schmidt Decomposition	126
5.13	Entropy of Entanglement with <code>tensors</code>	129
5.14	Schmidt Basis with <code>tensors</code>	130
5.15	Product States and Maximally Entangled States with <code>tensors</code>	134
5.16	Entanglement in the Two-Qubit TIM	136
5.16.1	$h_0 = h$ and $h_1 = 0$	138
5.17	Further Reading	139
	References	139
6	Variational Algorithms, Quantum Approximate Optimization Algorithm, and Neural Network Quantum States with Two Qubits	141
6.1	Introduction	141
6.2	Training the Two-Qubit Transverse-Field Ising Model	141
6.3	Training with Entanglement	150
6.4	The Quantum Approximate Optimization Algorithm	154

6.5	Neural Network Quantum States	159
6.6	Further Reading	167
	References	167
7	Phase Space Representation	169
7.1	Introduction	169
7.2	The Characteristic Function and Operator Ordering	170
7.3	The Characteristic Function in Terms of Real Variables	171
7.4	Gaussian States	175
7.4.1	Vacuum State	176
7.4.2	Coherent State	177
7.4.3	Thermal State	177
7.4.4	Proof of Eq. (7.42)	178
7.5	Covariance Matrix in Terms of the Derivatives of χ	179
7.5.1	Proof of Eqs. (7.72) and (7.73) for General States*	180
7.5.2	Proof of Eqs. (7.72) and (7.73) for a Gaussian State* ..	182
7.6	Covariance Matrices and Uncertainties	182
7.6.1	The Permuted Covariance Matrix	184
7.6.2	Ladder Operators and Complex Covariance matrix	187
7.7	Gaussian Characteristic Function	188
7.7.1	Remarks on the shape of a Vector	190
7.8	Linear Transformations and Symplectic Matrices	191
7.8.1	Proof of Eqs. (7.141) and (7.142)*	192
7.9	The $\mathbf{U}$ and $\mathbf{M}$ Matrices*	193
7.9.1	Coding the Matrices $\mathbf{R}_p$ and $\mathbf{R}_q$	197
7.10	Generating a Symplectic Matrix for a Random Gate	198
7.11	Further Reading	199
	References	200
8	States as a Neural Networks and Gates as Pullbacks	201
8.1	Introduction	201
8.2	The Simplest Neural Network for a Gaussian State	201
8.3	Gaussian Neural Network with Bias Input	202
8.4	The Vacuum Layer	205
8.5	Building a Model and the “Bug Train”	206
8.6	Pullback	207
8.7	The Pullback Layer	208
8.8	Pullback of Gaussian States	209
8.9	Coding the Linear Layer	210
8.10	Pullback Cascading	211
8.11	The Glauber Displacement Layer	213
8.12	A Neural Network Representation of a Coherent State	214
8.13	A Linear Layer for a Random Interferometer	216
	Reference	218

9	Quantum Reservoir Computing	219
9.1	Introduction	219
9.2	Observable Quantities as Derivatives of χ	219
9.3	A Coherent State in a Random Interferometer	221
9.4	Training a Complex Medium for an Arbitrary Coherent State	223
9.5	Training to Fit a Target Characteristic Function	225
9.6	Training by First Derivatives	228
9.7	Training by Second Derivatives	230
9.7.1	Proof of Eq. (9.5)*	232
9.8	Two Trainable Interferometers and a Reservoir	233
9.9	Phase Modulator	234
9.10	Training Phase Modulators	237
9.11	Further Reading	238
	References	238
10	Squeezing, Beam Splitters, and Detection	239
10.1	Introduction	239
10.2	The Generalized Symplectic Operator	239
10.3	Single-Mode Squeezed State	240
10.4	Multimode Squeezed Vacuum Model	241
10.5	Covariance Matrix and Squeezing	244
10.6	Squeezed Coherent States	245
10.6.1	Displacing the Squeezed Vacuum	245
10.6.2	Squeezing the Displaced Vacuum	246
10.7	Two-Mode Squeezing Layer	248
10.8	Beam Splitter	252
10.9	The Beam Splitter Layer	253
10.10	Photon Counting Layer	256
10.11	Homodyne Detection	260
10.12	Measuring the Expected Value of the Quadrature Operator	263
	References	264
11	Uncertainties and Entanglement	265
11.1	Introduction	265
11.2	The HeisenbergLayer for General States	266
11.2.1	The LaplacianLayer	267
11.2.2	The BiharmonicLayer	268
11.2.3	The HeisenbergLayer	271
11.3	Heisenberg Layer for Gaussian States	273
11.4	Testing the HeisenbergLayer with a Squeezed State	276
11.5	Proof of Eqs. (11.4) and (11.5)* and (11.9)*	276
11.6	DifferentialGaussianLayer	281
11.7	Uncertainties in Homodyne Detection	281
11.7.1	Proof of Eqs. (11.39) and (11.40)*	282
11.8	Uncertainties for Gaussian States	283
11.9	DifferentialGaussianLayer on Coherent States	287

11.10	DifferentialGaussianLayer in Homodyne Detection ..	289
11.11	Entanglement with Gaussian States	291
11.12	Beam Splitters as Entanglers	292
11.13	Entanglement by the Reduced Characteristic Function	293
11.14	Computing the Entanglement	295
11.15	Training the Model to Maximize the Entanglement.....	298
11.16	Further Reading	299
	References	299
12	Gaussian Boson Sampling	301
12.1	Introduction	301
12.2	Boson Sampling in a Single Mode	302
12.3	Boson Sampling with Many Modes	303
12.4	Gaussian States.....	305
12.5	Independent Coherent States	306
12.6	Zero Displacement Case in Complex Variables and the Hafnian	307
	12.6.1 The Resulting Algorithm	309
	12.6.2 Proof of Eq. (12.39)*	310
12.7	The Q-Transform	311
12.8	The Q-Transform Layer	313
12.9	The Multiderivative Operator	315
12.10	Single-Mode Coherent State	317
12.11	Single-Mode Squeezed Vacuum State	319
12.12	Multimode Coherent Case	321
12.13	A Coherent Mode and a Squeezed Vacuum	323
12.14	A Squeezed Mode and a Coherent Mode in a Random Interferometer	325
12.15	Gaussian Boson Sampling with Haar Random Unitary Matrices	326
	12.15.1 The Haar Random Layer	327
	12.15.2 A Model with a Varying Number of Layers	329
	12.15.3 Generating the Sampling Patterns.....	330
	12.15.4 Computing the Pattern Probability	332
12.16	Training Boson Sampling	333
12.17	The Loss Function	335
12.18	Trainable GBS Model	335
12.19	Boson Sampling the Model.....	336
12.20	Training the Model.....	341
12.21	Further Reading	345
	References	345
13	Variational Circuits for Quantum Solitons	347
13.1	Introduction	347
13.2	The Bose-Hubbard Model	348

13.3	Ansatz and Quantum Circuit	349
13.3.1	Proof of Eq. (13.7)*	351
13.3.2	Proof of Eq. (13.8)*	353
13.4	Total Number of Particles in Real Variables	353
13.5	Kinetic Energy in Real Variables	353
13.6	Potential Energy in Real Variables	354
13.7	Layer for the Particle Number	354
13.8	Layer for the Kinetic Energy	357
13.9	Layer for the Potential Energy	359
13.10	Layer for the Total Energy	360
13.11	The Trainable Boson Sampling Ansatz	361
13.12	Connecting the BSVa to the Measurement Layers	362
13.13	Training for the Quantum Solitons	363
13.14	Entanglement of the Single Soliton	367
13.15	Entangled Bound States of Solitons	369
13.16	Boson Sampling	371
13.17	Conclusion	372
13.18	Further Reading	372
	References	373
	Index	375