

Patrizia Pucci
James Serrin

The Maximum Principle

Birkhäuser
Basel · Boston · Berlin

Authors:

Patrizia Pucci
Dipartimento di Matematica e Informatica
Università degli Studi di Perugia
Via Vanvitelli 1
06123 Perugia
Italy
pucci@dipmat.unipg.it

James Serrin
University of Minnesota
Department of Mathematics
Minneapolis, MN 55455
USA
e-mail: serrin@math.umn.edu

2000 Mathematics Subject Classification 35J15, 35J60, 35J70, 35A05, 35B05, 35B50,
35R45, 58J70.

Library of Congress Control Number: 2007929013

Bibliographic information published by Die Deutsche Bibliothek
Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie;
detailed bibliographic data is available in the Internet at <<http://dnb.ddb.de>>.

ISBN 978-3-7643-8144-8 Birkhäuser Verlag AG, Basel · Boston · Berlin

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, broadcasting, reproduction on microfilms or in other ways, and storage in data banks. For any kind of use whatsoever, permission from the copyright owner must be obtained.

© 2007 Birkhäuser Verlag AG
Basel · Boston · Berlin
P.O. Box 133, CH-4010 Basel, Switzerland
Part of Springer Science+Business Media
Printed on acid-free paper produced from chlorine-free pulp. TCF∞
Printed in Germany

ISBN 978-3-7643-8144-8

e-ISBN 978-3-7643-8145-5

9 8 7 6 5 4 3 2 1

www.birkhauser.ch

Contents

Preface	ix
1 Introduction and Preliminaries	
1.1 Introduction	1
1.2 Notation	10
2 Tangency and Comparison Theorems for Elliptic Inequalities	
2.1 The contributions of Eberhard Hopf	13
2.2 Tangency and comparison principles for quasilinear inequalities	21
2.3 Maximum and sweeping principles for quasilinear inequalities	25
2.4 Comparison theorems for divergence structure inequalities	30
2.5 Tangency theorems via Harnack's inequality	34
2.6 Uniqueness of the Dirichlet problem	37
2.7 The boundary point lemma	39
2.8 Appendix: Proof of Eberhard Hopf's maximum principle	42
Notes	46
Problems	46
3 Maximum Principles for Divergence Structure Elliptic Differential Inequalities	
3.1 Distribution solutions	51
3.2 Maximum principles for homogeneous inequalities	54
3.3 A maximum principle for thin sets	59

3.4 A comparison theorem in $W^{1,p}(\Omega)$	61
3.5 Comparison theorems for singular elliptic inequalities	62
3.6 Strongly degenerate operators	68
3.7 Maximum principles for non-homogeneous elliptic inequalities	72
3.8 Uniqueness of the singular Dirichlet problem	78
3.9 Appendix: Sobolev's inequality	79
Notes	81
Problems	81
4 Boundary Value Problems for Nonlinear Ordinary Differential Equations	
4.1 Preliminary lemmas	83
4.2 Existence theorems	89
4.3 Existence theorems on a half-line	92
4.4 The end point lemma	96
4.5 Appendix: Proof of Proposition 4.2.1	97
Problems	101
5 The Strong Maximum Principle and the Compact Support Principle	
5.1 The strong maximum principle	103
5.2 The compact support principle	105
5.3 A special case	107
5.4 Strong maximum principle: Generalized version	110
5.5 A boundary point lemma	119
5.6 Compact support principle: Generalized version	120
Notes	125
Problems	126
6 Non-homogeneous Divergence Structure Inequalities	
6.1 Maximum principles for structured inequalities	127
6.2 Proof of Theorems 6.1.1 and 6.1.2	131
6.3 Proof of Theorem 6.1.3 and the first part of Theorem 6.1.5	139
6.4 Proof of Theorem 6.1.4 and the second part of Theorem 6.1.5	142

6.5 The case $p = 1$ and the mean curvature equation	146
Notes	150
Problems	150
7 The Harnack Inequality	
7.1 Local boundedness and the weak Harnack inequality	153
7.2 The Harnack inequality	163
7.3 Hölder continuity	166
7.4 The case $p \geq n$	171
7.5 Appendix. The John–Nirenberg theorem	173
Notes	179
Problems	180
8 Applications	
8.1 Cauchy–Liouville Theorems	181
8.2 Radial symmetry	186
8.3 Symmetry for overdetermined boundary value problems	195
8.4 The phenomenon of dead cores	203
8.5 The strong maximum principle for Riemannian manifolds	218
Problems	220
Bibliography	223
Subject Index	233
Author Index	235