Table 1. Natural Trigonometric Functions

Angle					Angle				
Degree	Radian	Sine	Cosine	Tangent	Degree	Radian	Sine	Cosine	Tangent
0°	0.000	0.000	1.000	0.000					
1°	0.017	0.017	1.000	0.017	46°	0.803	0.719	0.695	1.036
2°	0.035	0.035	0.999	0.035	47°	0.820	0.731	0.682	1.072
3°	0.052	0.052	0.999	0.052	48°	0.838	0.743	0.669	1.111
4°	0.070	0.070	0.998	0.070	49°	0.855	0.755	0.656	1.150
5°	0.087	0.087	0.996	0.087	50°	0.873	0.766	0.643	1.192
6°	0.105	0.105	0.995	0.105	51°	0.890	0.777	0.629	1.235
7°	0.122	0.122	0.993	0.123	52°	0.908	0.788	0.616	1.280
8°	0.140	0.139	0.990	0.141	53°	0.925	0.799	0.602	1.327
9°	0.157	0.156	0.988	0.158	54°	0.942	0.800	0.588	1.376
10°	0.175	0.174	0.985	0.176	55°	0.960	0.819	0.574	1.428
11°	0.192	0.191	0.982	0.194	56°	0.977	0.829	0.559	1.483
12°	0.209	0.208	0.978	0.213	57°	0.995	0.839	0.545	1.540
13°	0.227	0.225	0.974	0.231	58°	1.012	0.848	0.530	1.600
14°	0.244	0.242	0.970	0.249	59°	1.030	0.857	0.515	1.664
15°	0.262	0.259	0.966	0.268	60°	1.047	0.866	0.500	1.732
16°	0.279	0.276	0.961	0.287	61°	1.065	0.875	0.485	1.804
17°	0.297	0.292	0.956	0.306	62°	1.082	0.883	0.469	1.881
18°	0.314	0.309	0.951	0.325	63°	1.100	0.891	0.454	1.963
19°	0.332	0.326	0.946	0.344	64°	1.117	0.899	0.438	2.050
20°	0.349	0.342	0.940	0.364	65°	1.134	0.906	0.423	2.145
21°	0.367	0.358	0.934	0.384	66°	1.152	0.914	0.407	2.246
22°	0.384	0.375	0.927	0.404	67°	1.169	0.921	0.391	2.356
23°	0.401	0.391	0.921	0.424	68°	1.187	0.927	0.375	2.475
24°	0.419	0.407	0.914	0.445	69°	1.204	0.934	0.358	2.605
25°	0.436	0.423	0.906	0.466	70°	1.222	0.940	0.342	2.748
26°	0.454	0.438	0.899	0.488	71°	1.239	0.946	0.326	2.904
27°	0.471	0.454	0.891	0.510	72°	1.257	0.951	0.309	3.078
28°	0.489	0.469	0.883	0.532	73°	1.274	0.956	0.292	3.271
29°	0.506	0.485	0.875	0.554	74°	1.292	0.961	0.276	3.487
30°	0.524	0.500	0.866	0.577	75°	1.309	0.966	0.259	3.732
31°	0.541	0.515	0.857	0.601	76°	1.326	0.970	0.212	4.011
32°	0.559	0.530	0.848	0.625	77°	1.344	0.974	0.225	4.332
33°	0.576	0.545	0.839	0.649	78°	1.361	0.978	0.208	4.705
34°	0.593	0.559	0.829	0.675	79°	1.379	0.982	0.191	5.145
35°	0.611	0.574	0.819	0.700	80°	1.396	0.985	0.174	5.671
36°	0.628	0.588	0.809	0.727	81°	1.414	0.988	0.456	6.314
37°	0.646	0.602	0.799	0.754	82°	1.431	0.990	0.139	7.115
38°	0.663	0.616	0.788	0.781	83°	1.449	0.993	0.122	8.144
39°	0.681	0.629	0.777	0.810	84°	1.466	0.995	0.105	9.514
40°	0.698	0.643	0.766	0.839	85°	1.484	0.996	0.087	11.43
41°	0.716	0.656	0.755	0.869	86°	1.501	0.998	0.070	14.30
42°	0.733	0.669	0.743	0.900	87°	1.518	0.999	0.052	19.08
43°	0.750	0.682	0.731	0.933	88°	1.536	0.999	0.035	28.64
44°	0.768	0.695	0.719	0.966	89°	1.553	1.000	0.017	57.29
45°	0.785	0.707	0.707	1.000	90°	1.571	1.000	0.000	

Table 2. Exponential Functions

x	e^x	e^{-x}	x	e^x	e^{-x}
0.00	1.0000	1.0000	2.5	12.182	0.0821
0.05	1.0513	0.9512	2.6	13.464	0.0743
0.10	1.1052	0.9048	2.7	14.880	0.0672
0.15	1.1618	0.8607	2.8	16.445	0.0608
0.20	1.2214	0.8187	2.9	18.174	0.0550
0.25	1.2840	0.7788	3.0	20.086	0.0498
0.30	1.3499	0.7408	3.1	22.198	0.0450
0.35	1.4191	0.7047	3.2	24.533	0.0408
0.40	1.4918	0.6703	3.3	27.113	0.0369
0.45	1.5683	0.6376	3.4	29.964	0.0334
0.50	1.6487	0.6065	3.5	33.115	0.0302
0.55	1.7333	0.5769	3.6	36.598	0.0273
0.60	1.8221	0.5488	3.7	40.447	0.0247
0.65	1.9155	0.5220	3.8	44.701	0.0224
0.70	2.0138	0.4966	3.9	49.402	0.0202
0.75	2.1170	0.4724	4.0	54.598	0.0183
0.80	2.2255	0.4493	4.1	60.340	0.0166
0.85	2.3396	0.4274	4.2	66.686	0.0150
0.90	2.4596	0.4066	4.3	73.700	0.0136
0.95	2.5857	0.3867	4.4	81.451	0.0123
1.0	2.7183	0.3679	4.5	90.017	0.0111
1.1	3.0042	0.3329	4.6	99.484	0.0101
1.2	3.3201	0.3012	4.7	109.95	0.0091
1.3	3.6693	0.2725	4.8	121.51	0.0082
1.4	4.0552	0.2466	4.9	134.29	0.0074
1.5	4.4817	0.2231	5	148.41	0.0067
1.6	4.9530	0.2019	6	403.43	0.0025
1.7	5.4739	0.1827	7	1096.6	0.0009
1.8	6.0496	0.1653	8	2981.0	0.0003
1.9	6.6859	0.1496	9	8103.1	0.0001
2.0	7.3891	0.1353	10	22026	0.00005
2.1	8.1662	0.1225			
2.2	9.0250	0.1108			
2.3	9.9742	0.1003			
2.4	11.023	0.0907			

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Preface

Analytic geometry and calculus at a college or university almost always consists of a three-semester course. Typically, the first two semesters cover plane analytic geometry and the calculus of functions of one variable. The third semester usually deals with three-dimensional analytic geometry, partial differentiation, multiple integration, and a selection of other topics which depend on the book used. Some courses may even include a small amount of linear algebra. Most texts for such a three-semester sequence run to an unwieldy 1,000 pages or more.

We believe that an instructor can add a great deal of flexibility to the calculus program by separating the text materials used in the third semester from those used in the first year. Such a division makes for a greater choice in the selection of topics taken up in the third semester. Moreover, at many universities there is a fourth semester of analysis in the lower division program. In such a case it is desirable to have one book which carries through the entire year, as this text does.

In recent years the percentage of students who enter college after completing a year of calculus in high school has been increasing; by now, the number is substantial. These students, many of whom have taken the Advanced Placement program, have mastered the calculus of functions of one variable from a variety of texts and are ready to begin the third semester of calculus with analytic geometry with a text suited to their needs.

In the first five chapters in this book we present the material which is most frequently taught in the third semester of calculus. We suppose that the student has completed the usual two semesters of plane analytic geometry and one-variable calculus from any standard text. Chapters 6 through 10 provide additional material which can be used either to replace some of the traditional third-semester course or to fill out a fourth semester of analysis. The latter option would give students a thorough preparation for a junior-level course in real analysis.

One of the main features of our text is the flexibility which results from the relative independence of the chapters. For example, if an instructor wishes to teach Chapter 6 on Fourier series and if the students have already had the standard topics on infinite series which we present in Sections 1 through 10 of Chapter 3, then the instructor need only present the advanced material on uniform convergence of series in Sections 11, 12, and 13 as preparation for Fourier series. On the other hand, if the instructor chooses to skip Chapter 6, there is no inconvenience in presenting the remainder of the book.

We also wish to emphasize the flexibility of our treatment of both vector field theory and Green's and Stokes' theorems in Chapters 9 and 10. A minimum of preparation from Chapters 2, 4, and 5 is needed for this purpose. We first establish Green's theorem for simple domains, a result which is adequate for most applications. Here the presentation is quite elementary. Then we continue with a section on orientable surfaces, as well as proofs of Green's and Stokes' theorems, which use a partition of unity. The serious student will benefit greatly from these sections, since the methods we use are straightforward, detailed, and sufficiently general so that, for example, it can be shown that Cauchy's theorem for complex analytic functions in general domains is a corollary of Green's theorem.

Chapter 7, on the implicit function theorem and the inverse function theorem, provides an excellent preparation for those students who intend to go on in mathematics. However, it may be skipped with little or no inconvenience by those instructors who prefer to concentrate on the last two chapters of the text. Chapter 8, on differentiation under the integral sign and improper integrals, treats a useful topic, especially for those planning to work in applied mathematics or related fields of technology. It is worth noting that the material in Chapter 8 is seldom presented in texts at the lower division level. As with Chapter 7, the omission of this chapter will not affect the continuity of the remainder of the book.

Many students are not familiar with the simple properties of matrices and determinants. Also, they are usually not aware of Cramer's rule for solving m linear equation in n unknowns when m and n are different integers. In an appendix we provide an introduction to matrices and determinants sufficient to establish Cramer's rule. The instructor may wish to use this material as optional independent reading for those interested students who are unfamiliar with linear algebra. We include illustrative examples and exercises in this appendix so that a good student can easily learn the material without help.

Berkeley, California MURRAY H. PROTTER
October 1984

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