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Probability Theory

An Introductory Course

Translated from the Russian
by D. Haughton

With 14 Figures

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