Undergraduate Texts in Mathematics

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(continued after index)

William McGowen Priestley

Calculus: A Liberal Art

Second Edition

With 242 illustrations

Springer

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To Patten

What is the opposite of *two*?
A lonely me. A lonely you.

Even now there is a very wavering grasp of the true position of mathematics as an element in the history of thought. I will not go so far as to say that to construct a history of thought without profound study of the mathematical ideas of successive epochs is like omitting Hamlet from the play which is named after him. That would be claiming too much. But it is certainly analogous to cutting out the part of Ophelia. This simile is singularly exact. For Ophelia is quite essential to the play, she is very charming—and a little mad. Let us grant that the pursuit of mathematics is a divine madness of the human spirit, a refuge from the goading urgency of contingent happenings.

Alfred North Whitehead
from *Mathematics as an Element in the History of Thought*

Preface to the Second Edition

This second edition of *Calculus: An Historical Approach* has been slimmed down for use in a one-semester calculus course intended primarily for liberal arts students seeking to fulfil general education requirements in mathematics. Retitled *Calculus: A Liberal Art* to reflect this more specialized purpose, it includes everything from its predecessor that would normally be met in the first semester. New material has been added to give the instructor more freedom in determining the mathematical level at which to pitch the course and in choosing what emphasis to place upon historical and philosophical issues connected with the development of calculus and the nature of mathematics. Those who wish to place less emphasis upon such issues, for example, or who wish to discuss them only after first jumping into calculus as quickly as possible, can jump from Chapter 1 to Chapter 4, reviewing intermittently, as needed, topics from Chapters 2 and 3.

What emphasis should be placed upon writing in a course like this? The classical liberal arts included grammar, rhetoric, and logic—disciplines that are still related to our modern notion of a liberal arts education. Some of the exercises and problems in Chapter 2 are designed to reinforce this relation and to recall the larger kinship with mathematics that is often overlooked today. Even a failed attempt to teach writing skills in mathematics may have a beneficial result. Students who try to learn how to write mathematics may inadvertently learn how to *read* mathematics. Appendix 4, entitled "Clean Writing in Mathematics", may be useful in this connection.

All the material here could—in theory, at least—be presented early in secondary school following courses in algebra and geometry. The main

reason for delaying its study has to do with the question of mathematical maturity.* No use is made here of trigonometric, logarithmic, or exponential functions except in occasional optional material indicating how such functions can be handled.

A perceptive remark made by George Pólya suggests how we can simultaneously learn mathematics and learn "about" mathematics—i.e., about the nature of mathematics and how it is developed:

> *If the learning of mathematics reflects to any degree the invention of mathematics, it must have a place for guessing, for plausible inference.*

The reader will find plenty of opportunity here for guessing. The early chapters go at a gentle pace and invite the reader to enter into the spirit of the investigation. Exercises asking the reader to "make a guess" should be taken in this spirit—as simply an invitation to speculate about what is the likely truth in a given situation without feeling any pressure to guess "correctly". Readers will soon realize that a matter about which they are asked to guess will likely be a topic of serious discussion later on.

The last couple of full sections in each chapter, after the first, often include several exercises designated as optional. Sometimes they offer brief glimpses of deeper ideas of real analysis. Likewise, the latter problems in most problem sets at the ends of chapters are generally more demanding. Readers can omit these if they wish and still find it easy to go on to study the next chapter. This challenging material is included only in the hope that it may encourage some more ambitious students to continue their study of calculus at the next level. The final appendix, "From Freely Falling Bodies to Taylor series", is included solely for this reason.

I wish to thank Hardy Grant for generously offering to read early drafts of much of the new material and for giving me the benefit of his sound judgment. I am grateful also to Bill Imbornoni for smoothly overseeing Springer-Verlag's production of this second edition with the same care that Joyce Schanbacher bestowed upon its predecessor some twenty years ago.

W.M.P.

January, 1998
Sewanee

* See p. xi.

Preface to the First Edition

This book is for students being introduced to calculus, and it covers the usual topics, but its spirit is different from what might be expected. Though the approach is basically historical in nature, emphasis is put upon ideas and their place—not upon events and their dates. Its purpose is to have students to learn calculus first, and to learn incidentally something about the nature of mathematics.

Somewhat to the surprise of its author, the book soon became animated by a spirit of opposition to the darkness that separates the sciences from the humanities. To fight the spell of that darkness anything at hand is used, even a few low tricks or bad jokes that seemed to offer a slight promise of success. To lighten the darkness, to illuminate some of the common ground shared by the two cultures, is a goal that justifies almost any means. It is possible that this approach may make calculus more fun as well.

Whereas the close ties of mathematics to the sciences are well known, the ties binding mathematics to the humanities are rarely noticed. The result is a distorted view of mathematics, placing it outside the mainstream of liberal arts studies. This book tries to suggest gently, from time to time, where a kinship between mathematics and the humanities may be found.

There is a misconception today that mathematics has mainly to do with scientific technology or with computers, and is thereby unrelated to humanistic thought. One sees textbooks with such titles as *Mathematics for Liberal Arts Majors*, a curious phrase that seems to suggest that the liberal arts no longer include mathematics.

No discipline has been a part of liberal arts longer than mathematics.

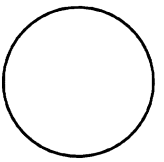
Three—logic, arithmetic, and geometry—of the original seven liberal arts are branches of mathematics. Plato's friend Archytas, who helped develop the whole idea of liberal education, was a distinguished mathematician. No true student of liberal arts can neglect mathematics.

How did it happen that mathematics, in the public eye, became dissociated from the humanities? In brief, the emergence and growth of scientific knowledge in the seventeenth century led to a polarization in academic circles. Science went one way, the humanities went another. Mathematics, at first in the middle, seems now to be more commonly identified with the sciences and with the technology they engendered.

Today in some academic institutions the state is not healthy. The ground between the sciences and the humanities is so dark that many well-meaning members on each side lack the education to see the most valuable contributions of the other. To the disadvantage of students, this is sometimes the case even among the faculties of so-called "liberal arts" colleges.

In the seventeenth century mathematics was a bridge between the two kinds of knowledge. Thus, for example, Isaac Newton's new physics could be read by Voltaire, who was at home both with Homer and with Archimedes. Voltaire even judged Archimedes to be superior, in imagination, to Homer.

The unity of knowledge which seemed attainable in the seventeenth century, and which has long been an ideal of liberal education, is still worth seeking. Today as in the time of Voltaire, and in the time of Plato, mathematics calls us to eye this goal.



For Anyone Afraid of Mathematics

Maturity, it has been said, involves knowing when and how to delay succumbing to an urge, in order by doing so to attain a deeper satisfaction. To be immature is to demand, like a baby, the immediate gratification of every impulse.

Perhaps happily, none of us is mature in every respect. Mature readers of poetry may be immature readers of mathematics. Statesmen mature in diplomacy may act immaturely in dealing with their own children. And mature mathematicians may on occasion act like babies when asked to listen to serious music, to study serious art, or to read serious poetry.

What is involved in many such cases is how we control our natural urge to get directly to the point. In mathematics, as in serious music or literature, the point sometimes simply cannot be attained immediately, but only by indirection or digression.

The major prerequisite for reading this book is a willingness to cultivate some measure of maturity in mathematics. If you get stuck, be willing to forge ahead, with suspended disbelief, to see where the road is leading. "Go forward, and faith will follow!" was d'Alembert's advice in the eighteenth century to those who would learn the calculus. Your puzzlement may vanish upon turning a page.

All that will be assumed at the outset is a nodding acquaintance with some elementary parts of arithmetic, algebra, and geometry, most of which was developed long before A.D. 1600. There will be some review in the early chapters, offering us as well a chance to outline the early history of mathematics.

I wish to thank Mary Priestley for helping me in this enterprise and for sharing with me its ups and downs. I am grateful also to Paul Halmos for his interest and encouragement.

W.M.P.

May, 1978
Sewanee

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Anecdote of the Jar

Wallace Stevens

I placed a jar in Tennessee
And round it was, upon a hill.
It made the slovenly wilderness
Surround that hill.

The wilderness rose up to it,
And sprawled around, no longer wild.
The jar was round upon the ground
And tall and of a port in air.

It took dominion everywhere.
The jar was gray and bare.
It did not give of bird or bush,
Like nothing else in Tennessee.