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Elementary Differential Geometry

Second Edition

Springer

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Preface

The Differential Geometry in the title of this book is the study of the geometry of curves and surfaces in three-dimensional space using calculus techniques. This topic contains some of the most beautiful results in Mathematics, and yet most of them can be understood without extensive background knowledge. Thus, for virtually all of this book, the only pre-requisites are a good working knowledge of Calculus (including partial differentiation), Vectors and Linear Algebra (including matrices and determinants).

Many of the results about curves and surfaces that we shall discuss are prototypes of more general results that apply in higher-dimensional situations. For example, the Gauss–Bonnet theorem, treated in Chapter 11, is the prototype of a large number of results that relate 'local' and 'global' properties of geometric objects. The study of such relationships formed one of the major themes of 20th century Mathematics.

We want to emphasise, however, that the *methods* used in this book are *not* necessarily those which generalise to higher-dimensional situations. (For readers in the know, there is, for example, no mention of 'connections' in the remainder of this book.) Rather, we have tried at all times to use the simplest approach that will yield the desired results. Not only does this keep the pre-requisites to an absolute minimum, it also enables us to avoid some of the conceptual difficulties often encountered in the study of Differential Geometry in higher dimensions. We hope that this approach will make this beautiful subject accessible to a wider audience.

It is a cliché, but true nevertheless, that Mathematics can be learned only by doing it, and not just by reading about it. Accordingly, the book contains over 200 exercises. Readers should attempt as many of these as their stamina permits. Full solutions to all the exercises are given at the end of the book, but these should be consulted only after the reader has obtained his or her own solution, or in case of desperation. We have tried to minimise the number of instances of the latter by including hints to many of the less routine exercises.

Preface to the Second Edition

Few books get smaller when their second edition appears, and this is not one of those few. The largest addition is a new chapter devoted to hyperbolic (or non-Euclidean) geometry. Quite reasonably, most elementary treatments of this subject mimic Euclid's axiomatic treatment of ordinary plane geometry. A much quicker route to the main results is available, however, once the basics of the differential geometry of surfaces have been established, and it seemed a pity not to take advantage of it.

The other two most significant changes were suggested by commentators on the first edition. One was to treat the tangent plane more geometrically - this then allows one to define things like the first and second fundamental forms and the Weingarten map as geometric objects (rather than just as matrices). The second was to make use of parallel transport. I only partly agreed with this suggestion as I wanted to preserve the elementary nature of the book, but in this edition I have given a definition of parallel transport and related it to geodesics and Gaussian curvature. (However, for the experts reading this, I have stopped just short of introducing connections.)

There are many other smaller changes that are too numerous to list, but perhaps I should mention new sections on map-colouring (as an application of Gauss-Bonnet), and a self-contained treatment of spherical geometry. Apart from its intrinsic interest, spherical geometry provides the simplest 'non-Euclidean' geometry and it is in many respects analogous to its hyperbolic cousin. I have also corrected a number of errors in the first edition that were spotted either by me or by correspondents (mostly the latter).

For teachers thinking about using this book, I would suggest that there are now three routes through it that can be travelled in a single semester, terminating with *one* of chapters 11, 12 or 13, and taking in along the way the necessary basic material from chapters 1–10. For example, the new section on spherical geometry might be covered only if the final destination is hyperbolic geometry.

As in the first edition, solutions to all the exercises are provided at the end of the book. This feature was almost universally approved of by student commentators, and almost as universally disapproved of by teachers! Being one myself, I do understand the teachers' point of view, and to address it

I have devised a large number of new exercises that will be accessible online to all users of the book, together with a solutions manual for teachers, at www.springer.com.

I would like to thank all those who sent comments on the first edition, from beginning students through to experts - you know who you are! Even if I did not act on all your suggestions, I took them all seriously, and I hope that readers of this second edition will agree with me that the changes that resulted make the book more useful and more enjoyable (and not just longer).

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