

# Contents

Introduction	1
Notation and Terminology	19
CHAPTER 1. The Role of Reductive Groups in Invariant Theory	23
§1. Reductive groups and the generalized Hilbert's 14th problem	23
§2. Quasihomogeneous varieties of reductive groups and the original Hilbert's 14th problem	25
CHAPTER 2. Constructive Invariant Theory	29
§1. Formulation and reduction of the problem	29
§2. A bound on the degree of a system of parameters and the main theorem	33
§3. The radical of the ideal $I$ and the approach suggested by Dieudonné and Carrell	40
CHAPTER 3. The Degree of the Poincaré Series of the Algebra of Invariants and a Finiteness Theorem for Representations wit Free Algebra of Invariants	43
§1. The degree of the Poincaré series and a functional equation	43
§2. The zonohedron of weights	51
§3. Finiteness theorems	56
CHAPTER 4. Syzygies in Invariant Theory	61
§0. A description of the results and additional notation	61
§1. Monotonicity theorems	64
§2. Bounds on $\text{hd } k[V]^G$ for certain types of groups	68
§3. Estimating $\text{hd } k[V]^G$ with the aid of one-dimensional tori of $G$	76
§4. Majorizing theorems for multiplicities, generic stabilizers, and stability	82
§5. Torus $T$ for the classical simple groups of rank $\geq 2$	87
§6. Torus $T$ for the exceptional simple groups	94
§7. Proof of the main theorem: the first case	100
§8. Proof of the main theorem: the second case	104

§9. Proof of the main theorem: the third case	113
§10. Examples	119
CHAPTER 5. Representations with Free Modules of Covariants	127
§1. Connections with equidimensionality: finiteness theorems	127
§2. Classification and equivalent characterizations: Igusa's condition	135
CHAPTER 6. A Classification of Normal Affine Quasihomogeneous Varieties of $SL_2$	147
§1. Some general results and the beginning of classification	147
§2. The conclusion of classification	154
§3. Application: the structure of orbit closures in finite-dimensional rational $SL_2$ -modules	164
CHAPTER 7. Quasihomogeneous Curves, Surfaces, and Solids	167
§1. A classification of irreducible quasihomogeneous curves	167
§2. A classification of irreducible affine surfaces with algebraic groups of automorphisms acting transitively on the complement of a finite number of points	176
§3. A classification of irreducible affine solids with algebraic groups of automorphisms acting transitively on the complement of a finite number of points	181
Appendix	201
§1. Appendix to Chapter 1	201
§2. Appendix to Chapter 2	203
§3. Appendix to Chapter 3	204
§4. Appendix to Chapter 4	213
§5. Appendix to Chapter 5	216
§6. Appendix to Chapter 6	225
Bibliography	231
Subject Index	243