

# **Applied Mathematical Sciences Volume 7**

# Applied Mathematical Sciences

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A.C. Pipkin

# Lectures on Viscoelasticity Theory

Second Edition

With 17 Illustrations



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