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Allen C. Pipkin

A Course on Integral Equations

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Series Preface

Mathematics is playing an ever more important role in the physical and biological sciences, provoking a blurring of boundaries between scientific disciplines and a resurgence of interest in the modern as well as the classical techniques of applied mathematics. This renewal of interest, both in research and teaching, has led to the establishment of the series: *Texts in Applied Mathematics (TAM)* .

The development of new courses is a natural consequence of a high level of excitement on the research frontier as newer techniques, such as numerical and symbolic computer systems, dynamical systems, and chaos, mix with and reinforce the traditional methods of applied mathematics. Thus, the purpose of this textbook series is to meet the current and future needs of these advances and encourage the teaching of new courses.

TAM will publish textbooks suitable for use in advanced undergraduate and beginning graduate courses, and will complement the *Applied Mathematical Sciences (AMS)* series, which will focus on advanced textbooks and research level monographs.

Foreword

This book is based on a one-semester course for graduate students in the physical sciences and applied mathematics. No great mathematical background is needed, but the student should be familiar with the theory of analytic functions of a complex variable. Since the course is on problem-solving rather than theorem-proving, the main requirement is that the student should be willing to work out a large number of specific examples.

The course is divided about equally into three parts. The first part, on Fredholm and Hilbert–Schmidt theory, is mostly theoretical. The last two parts emphasize problem-solving, using complex variables. The solution of convolution equations by using one- and two-sided Laplace transforms is covered in the second part, and the third part deals with Cauchy principal value integrals.

The chapters on Fredholm theory and Hilbert–Schmidt theory stress the analogy with linear algebra, since that is the point of these subjects, and much of the discussion is presented in terms of finite-dimensional matrices and vectors. Students who understand these chapters should be able to read the more detailed treatment given by Riesz and Sz.-Nagy without getting lost.

In preparation for the material on convolution equations, there is a short introduction to the Laplace transform. For reference, this chapter also includes proofs of some of the theorems that are taken for granted in lectures. Many of these proofs are adapted from Widder's book on Laplace transforms.

The character of Laplace transforms as analytic functions is emphasized in the chapters on convolution equations. The use of analytic function theory continues in the final part of the course, which begins with a chapter on the evaluation of principal value integrals. Again for reference, this chapter also includes proofs of some of the theorems that are being used. Widder's book on integral transforms is a good source for such theorems.

The principal value equations that we consider always have some or all of the real axis as the integration contour. Understanding this case seems to be sufficient for understanding the results about arbitrary contours in Muskhelishvili's treatise, and in fact we can go a little further than Muskhelishvili in the matter of infinite contours.

The complex variable methods that are used in the last two-thirds of the course put the student in a position to understand solutions based on the Wiener–Hopf technique. There is only a relatively short chapter on Wiener–Hopf equations, giving a few examples in which the analysis is completely concrete.

I would like to thank Larry Sirovich for his encouragement in this project. Kate MacDougall deserves particular thanks for her skill and patience in preparing the manuscript.

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