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Associative Algebras

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Preface

For many people there is life after 40; for some mathematicians there is algebra after Galois theory. The objective of this book is to prove the latter thesis. It is written primarily for students who have assimilated substantial portions of a standard first year graduate algebra textbook, and who have enjoyed the experience. The material that is presented here should not be fatal if it is swallowed by persons who are not members of that group.

The objects of our attention in this book are associative algebras, mostly the ones that are finite dimensional over a field. This subject is ideal for a textbook that will lead graduate students into a specialized field of research. The major theorems on associative algebras include some of the most splendid results of the great heros of algebra: Wedderburn, Artin, Noether, Hasse, Brauer, Albert, Jacobson, and many others. The process of refinement and clarification has brought the proof of the gems in this subject to a level that can be appreciated by students with only modest background. The subject is almost unique in the wide range of contacts that it makes with other parts of mathematics. The study of associative algebras contributes to and draws from such topics as group theory, commutative ring theory, field theory, algebraic number theory, algebraic geometry, homological algebra, and category theory. It even has some ties with parts of applied mathematics.

There is no intention to make this book an encyclopedia of associative algebra. Such a book would be a useful research tool, but it would not fit the needs of a novice mathematician. On the other hand, it is more than a rehash of existing expositions of the theory of associative algebras. The classical results of the subject are explored more deeply than in most student-oriented expositions of associative algebras, and the recent developments in the theory of algebras are liberally sampled. The serious student will find a substantial variety of challenges and rewards in the book.

Roughly speaking, the book is divided into two parts. Part one occupies chapters one through eleven. It could be called "the classical theory of associative algebras." This first part contains the basic structure and representation theorems for associative algebras: Wedderburn's Structure Theorem for Semisimple Algebras, Wedderburn's Principal Theorem, the structure of projective modules of Artinian algebras, and the recent work on representation types. Part two of the book concentrates on central simple algebras. It is organized around the concept of the Brauer group of a field. Chapter 12 builds the tools that are needed to construct the edifice of central simple algebras: the Jacobson Density Theorem, the Noether–Skolem Theorem, and the Double Centralizer Theorem. The topics that part two covers are fairly traditional: splitting fields, cohomological characterization of the Brauer group, cyclic algebras, the reduced norm and its applications, the Brauer groups of local and global fields, and finally an introduction to Amitsur's work on generic algebras.

The difficulty level of the book is a piecewise increasing graph. Each chapter begins with elementary material and escalates in complexity. The last few sections of each chapter contain the specialized and (usually) more difficult topics. At the same time, the median difficulty level of the chapters follows an increasing curve. Probably the best advice for readers of the book is to start at the beginning and plod through it to the end.

Every section of the book is equipped with at least one exercise. The exercises are included for the usual reasons: to keep the serious students awake; to ease the author's conscience pangs over omitted proofs; and to include results for which there is no room in the text. Most of the exercises are of the "follow your nose" variety. The non-trivial problems are accompanied by generous hints. In fact, some of the hints are so extensive that they might justifiably be called proofs.

Following an established tradition, we conclude this preface with acknowledgments and thanks to the friends who supported the preparation of the book. A list of these persons should include the names of a couple of dozen listeners who endured the author's lectures at the University of Connecticut, the University of Arizona, and the University of Hawaii. Most of these people will remain anonymous, but special mention is due to Javier Gomez, Oma Hamara, Eliot Jacobson, Bill Ullery, Bill Velez, and Kwang-Shang Wong whose eagle eyes found some of the numerous errors in the preliminary manuscript. Chuck Vinsonhaler deserves particular recognition for using several parts of the book as a basis for his own lectures. His suggestions and corrections have been extremely valuable.

The majority of credit for the completion of this book is owed to Marilyn Pierce. It was her patience and impatience that kept the project moving from its beginning to the end. She typed, corrected, and recorrected the whole manuscript. Her help and encouragement were always given generously, even though she has long held the author's solemn written promise never to write another book. It is to Marilyn that this book is dedicated.

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