George M. Phillips

## Two Millennia of Mathematics From Archimedes to Gauss



George M. Phillips Mathematical Institute University of St. Andrews St. Andrews KY16 9SS Scotland

Editors-in-Chief Rédacteurs-en-chef Jonathan M. Borwein Peter Borwein Centre for Experimental and Constructive Mathematics Department of Mathematics and Statistics Simon Fraser University Burnaby, British Columbia V5A 1S6 Canada

Mathematics Subject Classification (2000): 00A05, 01A05

Library of Congress Cataloging-in-Publication Data Phillips, G.M. (George McArtney) Two millennia of mathematics : from Archimedes to Gauss / George M. Phillips. p. cm. — (CMS books in mathematics ; 6) Includes bibliographical references and index. ISBN 978-1-4612-7035-5 ISBN 978-1-4612-1180-8 (eBook) DOI 10.1007/978-1-4612-1180-8 1. Mathematics—Miscellanea. 2. Mathematics—History. I. Title. II. Series. QA99 .P48 2000 510—dc21 00-023807

Printed on acid-free paper.

© 2000 Springer Science+Business Media New York Originally published by Springer-Verlag New York,Inc. in 2000 Softcover reprint of the hardcover 1st edition 2000

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer Science+Business Media, LLC), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden. The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

Production managed by Timothy Taylor; manufacturing supervised by Erica Bresler. Photocomposed copy prepared from the author's  $IaT_{E}X$  files.

ISBN 978-1-4612-7035-5

SPIN 10762921

## Contents

Preface					
A	cknov	wledgments	ix		
1	From Archimedes to Gauss				
	1.1	Archimedes and Pi	2		
	1.2	Variations on a Theme	8		
	1.3	Playing a Mean Game	18		
	1.4	Gauss and the AGM	33		
2	Logarithms				
	2.1	Exponential Functions	45		
	2.2	Logarithmic Functions	49		
	2.3	Napier and Briggs	60		
	2.4	The Logarithm as an Area	72		
	2.5	Further Historical Notes	76		
3	Interpolation 81				
	3.1	The Interpolating Polynomial	81		
	3.2	Newton's Divided Differences	88		
	3.3	Finite Differences	93		
	3.4	Other Differences	98		
	3.5	· · · · · · · · · · · · · · · · · · ·	105		
	3.6	The Neville–Aitken Algorithm	115		

## xii Contents

	3.7	Historical Notes	119		
4	Continued Fractions				
	4.1	The Euclidean Algorithm	121		
	4.2	Linear Recurrence Relations	131		
	4.3	Fibonacci Numbers	138		
	4.4	Continued Fractions	147		
	4.5	Historical Notes	161		
5	More Number Theory				
	5.1	The Prime Numbers	166		
	5.2	Congruences	172		
	5.3	Quadratic Residues	181		
	5.4	Diophantine Equations	188		
	5.5	Algebraic Integers	194		
	5.6	The equation $x^3 + y^3 = z^3$	204		
	5.7	Euler and Sums of Cubes	209		
Re	References				
Index					